Transport via a Quantum Shuttle

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Abstract

We investigate the effect of quantisation of vibrational modes on a system in which the transport path is through a quantum dot mounted on a cantilever or spring such that tunnelling to and from the dot is modulated by the oscillation. We consider here the implications of quantum aspects of the motion. Peaks in the current voltage characteristic are observed which correspond to avoided level crossings in the eigenvalue spectrum. Transport occurs through processes in which phonons are created. This provides a path for dissipation of energy as well as a mechanism for driving the oscillator, thus making it easier for electrons to tunnel onto and off the dot and be ferried across the device.

Key words: nanotechnology; mesoscopics; quantum shuttle; NEMS

Recent advances in the fabrication of nanomechanical devices[1] are showing a distinct trend towards systems in which a quantum description is required not only for the electronic behaviour but also for the mechanical aspects. We therefore consider a model nano–electro–mechanical system consisting of a quantum dot attached to a spring or cantilever which moves between 2 contacts; thus acting as an electron shuttle (Fig. 1). Devices of this sort have previously been fabricated albeit with classical mechanical behaviour[2].

Fig. 1. A quantum shuttle consisting of a dot on springs flanked by 2 stationary dots attached to semi–infinite leads.

In this work we consider 2 models:
(i) a 3 dot model in which the moveable dot is flanked by 2 stationary dots. This is described by a tight–binding model[3].

$$H = \varepsilon_l |l\rangle\langle l| + \varepsilon_r |r\rangle\langle r| + \varepsilon_c(\hat{x})|c\rangle\langle c| + \hbar\omega\hat{d}^{\dagger}\hat{d} \quad (1)$$
$$- V e^{-\alpha(\hat{d}^{\dagger}+\hat{d})}(|c\rangle\langle l| + |l\rangle\langle c|)$$
$$- V e^{\alpha[(\hat{d}^{\dagger}+\hat{d})-2x_0]}(|c\rangle\langle r| + |r\rangle\langle c|).$$

where $|i\rangle\langle i|$ are projection operators for the three electronic states and the vibrational mode, frequency ω, is operated on by $\hat{d}$.

(ii) a scattering model in which the dot is embedded between semi–infinite leads in a Landauer[4] geometry.

$$H = -E_e \frac{\partial^2}{\partial x_e^2} - E_s \frac{\partial^2}{\partial y_s^2} - \tfrac{1}{4}y_s^2 \quad (2)$$
$$+ V_1 \left[\Xi(x_e, d_c) - \Xi(x_e - s y_s, d_s) \right]$$

where x_e and y_s represent the electron and phonon coordinates respectively, E_e and E_s the corresponding energy scales, s the shuttle displacement,

$$\Xi(x, d) = \Theta(x_e + \tfrac{1}{2}d) - \Theta(x_e - \tfrac{1}{2}d)$$

a barrier of width d; d_c and d_s represent the separation of the contacts and the size of the shuttle respectively.

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The presence of the exponential terms in (2) and a similar term in the matching conditions of (3) make the tunnelling rates sensitive to the position of the shuttle.
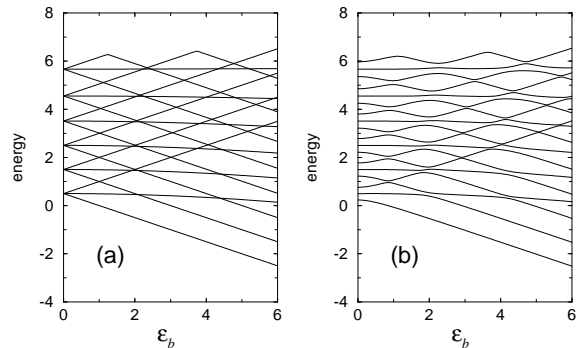

Fig. 2. Eigenvalues of the 3–dot model as a function of potential difference between the outer dots; (a) without tunnelling (b) with tunnelling

When the energies of the 3 dot system are plotted against the potential difference between the outer dots the tunnelling through the shuttle results in a series of anti-crossings (Fig. 2). These are of 2 types: those involving only the outer dots, such as at $(1, 1)$ in fig. 2; those involving all 3 dots, such as at $(2, 1.5)$ in fig. 2.
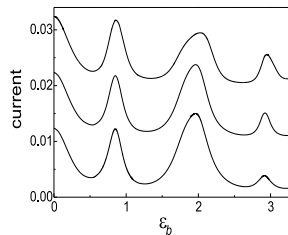

Fig. 3. Current in the 3–dot model as a function of potential difference for 3 different damping rates (shifted for clarity).

The resulting current–voltage characteristic is shown in fig. 3. There is a peak at $\epsilon_b = 0$ due to resonant tunnelling through all 3 dots. The other features may be associated with anti–crossings in fig. 2. The maximum at $\varepsilon_b \approx 0.8$ occurs when the difference between the energies of the left and right–hand dots differ by that of a single phonon, whereas the peak at $\varepsilon_b \approx 2.0$ involves 2 phonons and is associated with a 3–way anti–crossing. Note, in particular, that the latter peak is much more sensitive to the damping of the phonons than is the former. This is clearly due to the involvement of the state on the shuttle itself.

Results for the scattering model are illustrated in fig. 4. The various curves correspond to different states of the shuttle before the electron is scattered. Note that the half–width of the peaks settles down to about double the phonon energy, corresponding to a transmis-
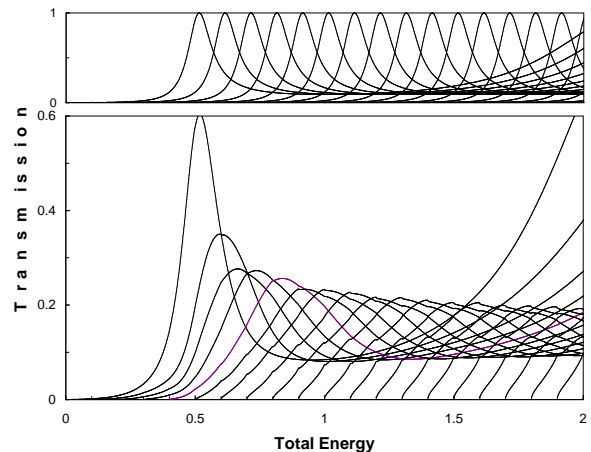

Fig. 4. Total transmission probability as a function of total energy (electron+phonon) for various incident phonon numbers ($n = 0 - 19$ start at E=n/10). Shuttle displacement is zero (upper figure), about 40% of the barrier width (lower figure). The onset of a 2nd resonant tunnelling peak is seen to the right.

sion time of about half the shuttle period as would be expected for the shuttle effect.

We have presented results for 2 different models of a quantum shuttle. In both cases there was no necessity to include a specific mechanism to drive the shuttle. As long as the potential difference across the system is greater than the phonon energy the shuttle may be driven by the creation of phonons. This in turn implies that the dissipation of this energy will play an important role in the behaviour of such a system.

Acknowledgements

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