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Conduction in a spatially varying magnetic field: one-dimensional strips, with zero mean field

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Abstract. We calculate the density of states and band conductivity of a two-dimensional electron gas with a periodically modulated magnetic field. We consider the case of strips where the magnetic field is alternately going up and down with zero mean. We find characteristic features in the density of states and the conductivity at low energies as the electrons are only able to drift parallel to the strips of magnetic field. As the energy is increased, and the electrons start to be able to drift perpendicular to the strips there are pronounced band-structure effects in the conductivity.

1. Introduction

The possibility of applying a spatially varying magnetic field to a two-dimensional electron gas (2DEG) has been considered recently by several authors [1–5]. Experiments with a magnetic field regularly modulated by strips of magnetic material, or of superconductor are expected to be practical. In this paper we analyse the conduction for one particular case: a magnetic field modulated periodically in one direction, with zero mean field. Unlike previous work this constitutes a form of strong modulation, since the variation of the field is large compared with its average, although, depending on the parameters, the cyclotron energy associated with the magnetic field may be small compared with the Fermi energy.

In section 2 we describe the Hamiltonian, and how, since the momentum parallel to the strips is conserved, the motion perpendicular to the strips is described by a simple one-dimensional Hamiltonian. In section 3 we discuss the classical motion of the electrons, and show how at low energies all the orbits drift parallel to the strips of the potential, whereas at higher energies the electrons can drift in any direction. In section 4 we present the results of a quantum mechanical calculation of the band conductivity. The perpendicular momentum dependent potential results in much stronger band effects being visible than for an electrostatic potential with the same effective potential for zero perpendicular momentum.

2. Effective potential

We consider a 2DEG with a magnetic field perpendicular to the sample of $B = B_0 \cos(2\pi x/a)$. In practice there will also be components of the magnetic field parallel to the 2DEG. The system can be described by the Hamiltonian

$$H = \frac{1}{2m}(p - eA)^2 \qquad (1)$$

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where $A = (aB_0/2\pi)(0, \sin(2\pi x/a), 0)$. The Hamiltonian is translationally invariant in the y direction, so that the momentum perpendicular to the modulation, p_y, is conserved, and the motion in the x direction is described by the one-dimensional Hamiltonian

$$H(x, p_x) = \frac{p_x^2}{2m} + \frac{1}{2m}\left(p_y - \frac{a}{2\pi}eB_0 \sin\left(\frac{2\pi x}{a}\right)\right)^2. \qquad (2)$$

The effective potential seen by an electron is shown in figure 1 for different values of the perpendicular momentum p_y. By way of contrast, recall what happens for a system in a uniform magnetic field: $A = B_0(0, y, 0)$ and the effective potential is a parabola, the position of the parabola's minimum depending on p_y. Here instead, with a zero mean field, the effective potential is periodic in x, and increasing the magnitude of p_y increases both the modulation and the mean value of the potential.

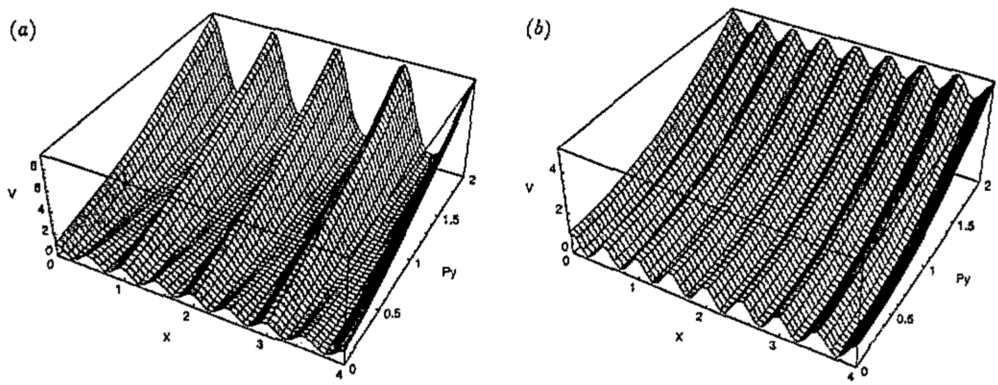

Figure 1. Effective potential as a function of position X and perpendicular momentum p_y. X is in units of the lattice periods, with the maximum of magnitude of the magnetic field at $X = 0, 0.5$ etc. p_y is in units of $aB_0/2\pi$ and the potential is in units of the threshold energy, $(1/2m)(aeB_0/2\pi)^2$. (a) Magnetic Hamiltonian, equation (2); (b) electrostatic Hamiltonian with same potential for $p_y = 0$.

3. Classical motion

Given the one-dimensional Hamiltonian in equation (2), the motion in the x direction is trivial: if the electron has a small momentum it oscillates in one of the minima; if it has a large enough momentum it travels with a steady mean velocity in the x direction. Figure 2 shows the possible motions for one particular origin. (Note that the dynamics are integrable because of the symmetry, and all the trajectories are periodic, modulo any uniform translation.)

The potential implies that there is a threshold energy,

$$E = \frac{1}{2m}\left(\frac{aeB_0}{2\pi}\right)^2 \qquad (3)$$

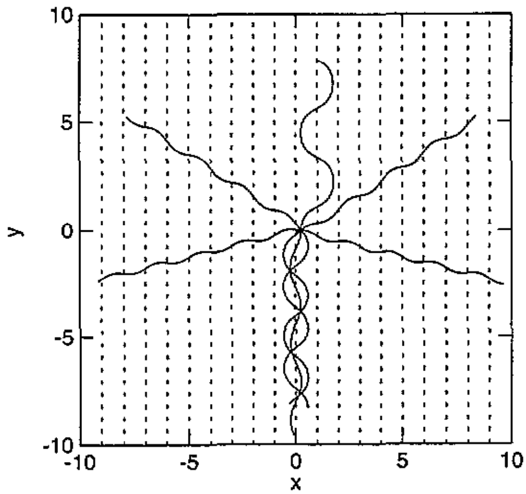

Figure 2. Paths of electron injected from a single point with the same speed, but with different initial directions. The dotted lines indicate the lines of zero magnetic field. The cyclotron radius at the points with the highest magnetic field is half the periodicity of the modulation.

below which there is no conduction perpendicular to the stripes, except by way of inelastic scattering, or scattering by impurities. Below this energy the only trajectories that exist are those where the electron drifts parallel to the stripes.

We have calculated numerically the conductivity perpendicular and parallel to the strips assuming a scattering time of 38 ps, equivalent to a mobility of the unmodulated 2DEG of $100 m^2 V^{-1} s^{-1}$. We have calculated the diffusion constant from individual trajectories of duration 38 ps as $D_{xx} = \langle \Delta x^2 \rangle / (2\tau)$, where Δx is the distance covered by the path in the x direction, and similarly for motion in the other direction. Doing this we neglect correlations between different paths, and any effect from the distribution of path lengths. Since $\omega_c \gg 1/\tau$ for the magnetic fields considered and the paths consist of straight lines with an oscillation superimposed we do not expect either of these simplifications to be significant. Since there is no net magnetic field in this problem there is no Hall resistance, $D_{xy} = 0$. For large magnetic fields such that the typical cyclotron radius of an electron is small compared with the lattice period there will be an enhancement to the conductivity parallel to the stripes because in this limit there are spatially separated regions where all the electrons drift in the same direction.

Figures 3 and 4 show the calculated conductivity parallel and perpendicular to the strip. We see that the conductivity parallel to the strips is only weakly affected by the presence of the magnetic field modulation. The conductivity perpendicular to the strips is greatly reduced. The threshold energy for drifting trajectories is marked on the figures. Below this energy the conduction perpendicular to strips arises from the scattering conductivity due to the oscillation of the electron about its mean trajectory.

4. Quantum mechanics

We now turn to what we would expect quantum-mechanically. For a given perpendicular wavevector the electron sees a periodic potential, given by equation (2), with Fourier components at $k_x = 2\pi/a$ and also because of the A^2 term at $k_x = 4\pi/a$. The size of the term at $k_x = 2\pi/a$ is seen to be proportional to k_y. This term vanishes for $k_y = 0$ so that the potential has half the periodicity at that point.

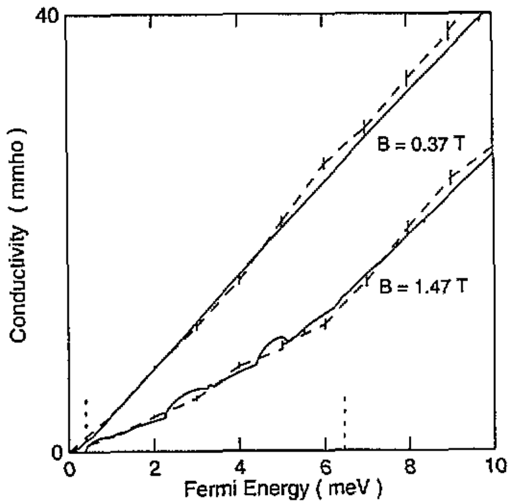

Figure 3. Conductivity parallel to the strips for two different magnetic fields. The solid line is the result of the quantum mechanical calculation of the band conductivity, the dashed line is the result of classical calculation. The dotted lines mark the threshold energy for drifting motion perpendicular to the strips for the two fields.

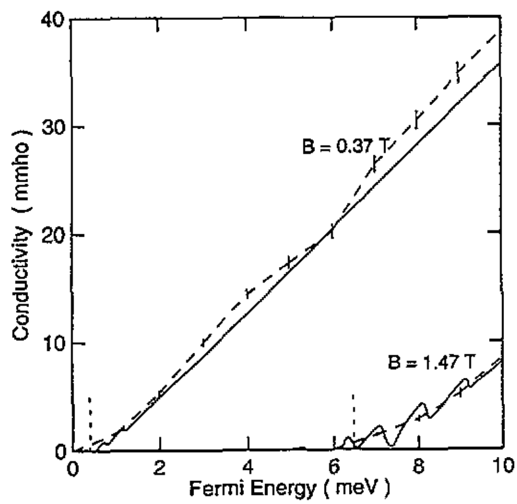

Figure 4. Conductivity perpendicular to the strips for two different magnetic fields. The solid line is the result of the quantum mechanical calculation of the band conductivity, the dashed line is the result of the classical calculation. The conductivity has been smoothed by the Fermi function for $T = 0.3$ K to remove the spurious oscillations associated with the finite number of k points used for the calculation. The dotted lines mark the threshold energy for drifting motion perpendicular to the strips for the two fields.

We have used the model and numerical techniques described in previous papers [6–8] to calculate the band conductivity and the density of states in this system. We have used the same parameters as for the classical calculation, with the addition of a quantum scattering time of $\tau_Q = 6.5$ ps for the density of states calculation.

Figure 3 shows the band conductivity parallel to the strips. The quantum mechanical behaviour is similar to the classical behaviour. The small systematic deviation at large energies is a consequence of the finite discretization used: the Hamiltonian for one unit cell was discretized on a 40×40 grid. For the weaker magnetic field shown the conductivity parallel to the strips shows little structure, except at the lowest energies. At the higher magnetic field we see more structure, which is associated with one-dimensional quantization. At low energies the electrons are confined to strips along the lines of zero magnetic field, and are only weakly coupled to adjacent strips by tunnelling. To a good approximation the electrons at low energy are therefore confined to a set of parallel, one-dimensional strips, and the structure seen in the conductivity arises from the quantized modes in these strips. At higher energies conduction is possible perpendicular to the strips, and the one-dimensional quantization disappears.

The conductivity perpendicular to the strips, shown in figure 4 is more affected by the modulation than the conductivity parallel to the stripes, as we expect. In particular we see some characteristic sharp dips in the conductance at certain energies. These are band-structure effects. These effects are more prominant than for a comparable electrostatic modulation, as found for weak modulation by Peeters and Vasilopoulos [4], because the effective potential grows and rises with increasing k_y rather than, just rising in energy as k_y increases, smearing out bandstructure effects, as happens for an electrostatic modulation. Comparing figures 3 and 4 we note how the effects of the one-dimensional quantization disappear as conduction perpendicular to the strips starts with increasing energy. We also see that in both cases the quantum mechanical and classical calculations give the same result, with the exception of some additional structure in the quantum case.

Figure 5 shows the density of states. In the region where the conductance is showing effects from the one-dimensional quantization, we see corresponding peaks in the density of states. At higher energies, where the electrons are free to drift perpendicular to the strips the structure in the density of states disappears. There is a double peak structure in the density of states, for example at approximately 0.45 T and 1.15 T. The lower peak is caused by electrons occupying the regions where the magnetic field is small. The upper peak arises as the electrons fill the regions of high magnetic field.

5. Summary

We have presented in this paper calculations for a novel, periodic, magnetically modulated system. Because the effective potential seen by the electron depends on its momentum perpendicular to the modulation, or more simply since the Lorentz force experienced by the electron increases with the velocity of the electron, band structure effects are more prominent than in a comparable electrostatically modulated system.

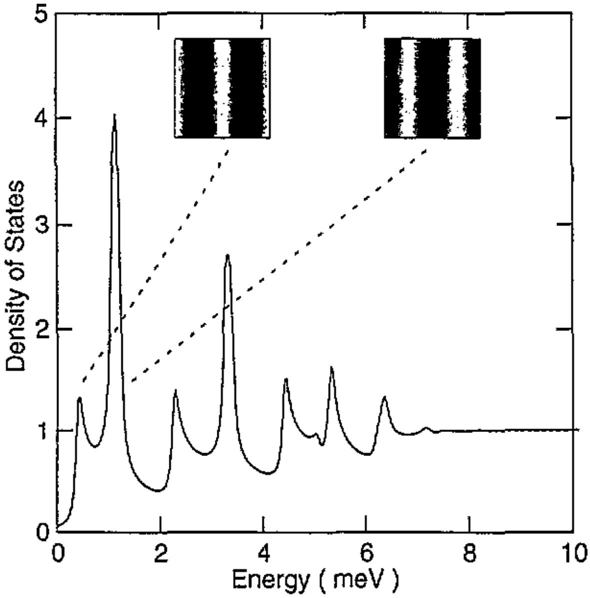

Figure 5. Density of states for a magnetic field of $B_0 = 1.47$ T. The local density of states at the two peaks indicated is shown in the insets. The darker areas indicate the higher density of states. The regions of high magnetic field run vertically at the edge and centre of the unit cell shown.

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