Group Theory

Problem Set 3 October 23, 2001

Note: Problems marked with an asterisk are for Rapid Feedback.

1.* List all of the subgroups of any group whose order is a prime number.

2.* Show that a group whose order is a prime number is necessarily cyclic, i.e., all of the elements can be generated from the powers of any non-unit element.

3. Suppose that, for a group G, $|G| = pq$, where p and q are both prime. Show that every proper subgroup of G is cyclic.

4.* Let g be an element of a finite group G. Show that $g^{|G|} = e$.

5. In a quotient group G/H, which set *always* corresponds to the unit "element"?

6. Show that, for an Abelian group, every element is in a class by itself.

7. Show that every subgroup with index 2 is self-conjugate.

Hint: The conjugating element is either in the subgroup or not. Consider the two cases separately.

8.* Consider the following cyclic group of order 4, $G = \{a, a^2, a^3, a^4 = e\}$ (cf. Problem 6, Problem Set 2). Show, by direct multiplication or otherwise, that the subgroup $H = \{e, a^2\}$ is self-conjugate and identify the elements in the factor group G/H.

9.* Suppose that there is an isomorphism ϕ from a group G onto a group G'. Show that the identity e of G is mapped onto the identity e' of G': $e' = \phi(e)$.

Hint: Use the fact that $e = ee$ must be preserved by ϕ and that $\phi(g) = e'\phi(g)$ for all g in G.