Group Theory

Problem Set 5 November 6, 2001

Note: Problems marked with an asterisk are for Rapid Feedback.

1. In proving Theorem 3.2, we established the relation $B_iB_i^\dagger = I$. Using the definitions in that proof, show that this result implies that $B_i^\dagger B_i = I$ as well.

Hint: Show that $B_iB_i^\dagger = I$ implies that $\tilde{A}_iD\tilde{A}_i^\dagger = D$.

2.* Consider the three-element group $G = \{e, a, b\}$ (Sec. 2.4).

(a) Show that this group is Abelian and cyclic (cf. Problem 2, Problem Set 3).

(b) Consider a one-dimensional representation based on choosing $a = z$, where z is a complex number. Show that for this to produce a representation of G, we must require that $z^3 = 1$.

(c) Use the result of (b) to obtain three representations of G. Given what you know about the irreducible representations of Abelian groups (Problem 8, Problem Set 4), are there any other irreducible representations of G?

3.* Generalize the result of Problem 2 to any cylic group of order n.

4.* Use Schur's First Lemma to prove that all the irreducible representations of an Abelian group are one-dimensional.

5.* Consider the following matrices:

$$e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad a = \tfrac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \quad b = \tfrac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$$

$$c = \tfrac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad d = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad f = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Verify that these matrices form a representation of S_3. Use Schur's first Lemma to determine if this representation reducible or irreducible. If reducible, determine the irreducible representations that are obtained from the diagonal form of these matrices.