Group Theory

Problem Set 7 November 20, 2001

Note: Problems marked with an asterisk are for Rapid Feedback.

1.* Identify the 12 symmetry operations of a regular hexagon.

2. Show that elements in the same class of a group must have the same order.

3.* Identify the 6 classes of this group.

Hint: You do not need to compute the conjugacy classes explicitly. Refer to the discussion for the group S_3 in Example 2.9 and use the fact that elements in the same class have the same order.

4.* How many irreducible representations are there and what are their dimensions?

5.* Construct the character table of this group by following the procedure outlined below:

(a) Enter the characters for the identical and "parity" representations. As in the case of S_3, the characters for the parity representation are either $+1$ or -1, depending on whether or not the the operation preserves the "handedness" of the coordinate system.

(b) Enter the characters for the "coordinate" representation obtained from the action on (x, y) for each group operation. Note that the character is the same for elements in the same class.

(c) Use the products $C_3 C_3^2 = E$ and $C_3^3 = E$ to identify the characters for all one-dimensional irreducible representations for the appropriate classes. The meaning of the notation C_n^m for rotations is discussed in Section 5.4.

(d) Use the result of (c) and the products $C_6 C_3 = C_2$ to deduce that the characters for the class of C_6 and those for the class of C_2 are the same. Then, use the orthogonality of the *columns* of the character table to compute these characters.

(e) Use the appropriate orthogonality relations for characters to compute the remaining entries of the character table.