Metals without Electrons: the Physics of Exotic Quantum Fluids

Derek K.K. Lee and Andrew J. Schofield

Blackett Laboratory, Imperial College, Prince Consort Rd, London SW7 2BW
School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT

Quantum complexity – the new frontier

The close of the nineteenth century and the close of the twentieth share a number of striking parallels. At the turn of the twentieth century, there was a feeling among some theoretical physicists that almost all that could be known was known and all that remained was details to be cleared up. These details appeared to be a number of minor discrepancies between the theory of classical physics and experiment. Classical physics had triumphed with the unification of electricity and magnetism, and J. J. Thomson's discovery of the electron had allowed him to develop a workable version of atomic physics. With the benefit of hindsight we now know that some of those "minor discrepancies" held the key to a revolution that transformed physics in the 1920's and has dramatically affected the technology of the twentieth century.

That revolution was the discovery of quantum mechanics. It utterly revised our perception of nature at the atomic level. The nineteenth-century view of physics (Newton's mechanics and Maxwell's electromagnetism) proved to be inadequate at these small scales. The pace of development since those days has proved so dramatic that the control of Thomson's electron at a quantum level is now possible. "Designer electronics" is commonplace in semiconductor technology from televisions and computers to mobile communications — quantum physics is working for us every day.

The successes born of physics in the early years of the twentieth century, quantum mechanics and Einstein's relativity, have left us with the perception that the challenges of theoretical physics lie at the extremes of length scale (see Figure 1). At one end, the study of the atom and quantum mechanics has progressed to the challenge of understanding how sub-nuclear particles interact at the smallest imaginable distances. Here, the excitement lies in the realm of string theory and exotic supersymmetries. At the other extreme, on the astronomical scale, the physics of black holes and the beginnings of the universe prove equally fascinating. Mirroring the end of the nineteenth century, some scientists feel that theoretical physics is almost complete. Indeed, the noted cosmologist Stephen Hawking entitled his inaugural lecture in Cambridge "The end of theoretical physics?" He concluded then (as now!) that the end was twenty years away.

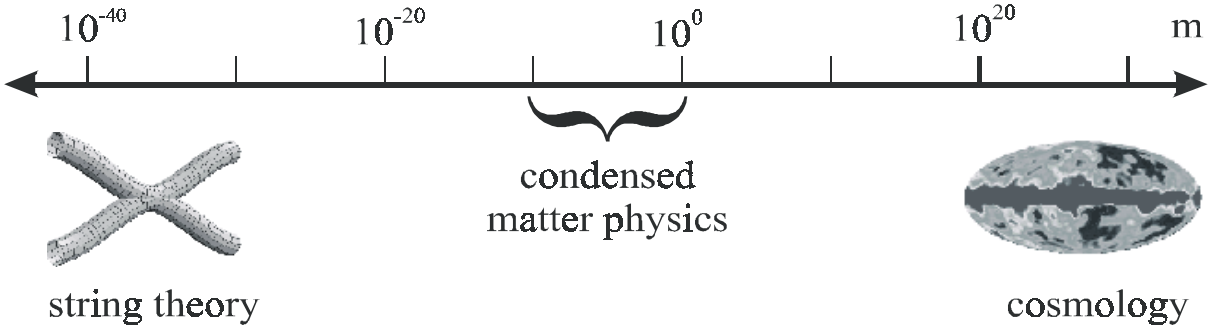

Figure 1: It is often tempting to view the challenges of theoretical physics in terms of the extremes of length. Condensed matter physics concerns itself with distances ranging from about one meter down to about one ten millionth of a millimeter. Compared to string theory at the very smallest lengths and cosmology at the size of the universe, condensed matter can seem rather mundane. However condensed matter physicists explore a new frontier — extreme complexity (see Figure 2).

However, a new dimension is emerging in physics that explores the physical world from an entirely different perspective. Measured on this alternative axis, we see that theoretical physics is far from over. On the contrary, there are the telltale signs that a completely new understanding is required to make progress here. Just as at the end of the nineteenth century, there are a growing number of "discrepancies" between theory and experiments, indicating something important to be discovered. This is a challenge every bit as fundamental to the physics at the extremes of length scale. It is a frontier that has implications for the technology of the twenty-first century, perhaps as great as those that have driven the semiconductor revolution. Those rising to this challenge are condensed matter physicists together with material scientists. The frontier we explore is that of extreme complexity (see Figure 2).

Condensed matter physics is the study of the atomic and electronic behaviour of the everyday matter around us — solids, liquids and gases. The laws of electromagnetism and quantum theory, which govern the behaviour of the electron and atoms, the basic ingredients of this matter, were developed in the 1920's. However the tremendous advances in material science have brought these ingredients together in new and complex ways. The new physics emerging from complex systems has shown us that merely knowing the basic constituents and their interactions can often be a poor guide to their collective properties. What we discover in more complex materials is rarely a simple extrapolation of the physics of simpler systems. Instead, we often find entirely new types of behaviour, which need new concepts to understand them. These "emergent phenomena" can be seen as we explore the axis of complexity (Figure 2). We may loosely parameterize the degree of complexity by the number of possible combinations of materials as we mix together more and more of the hundred or so elements of the periodic table. From the physics of the hydrogen atom we move in complexity to simple metals like niobium. There we see electrons pairing to form a superconductor. Mixing two elements together gives us, for example, gallium arsenide. In devices made from this we see the physics of the fractional quantum Hall effect where electrons bind to magnetic flux. The tertiary alloys of uranium often appear to be made of electrons with masses thousands of times greater than normal. The current state of the art in material science lies in the quaternary oxide materials including the `high temperature' superconductors. None of these discoveries could have been predicted on the basis of simpler systems. The science behind all of these emergent phenomena shows us that the whole is frequently greater than the sum of the parts.

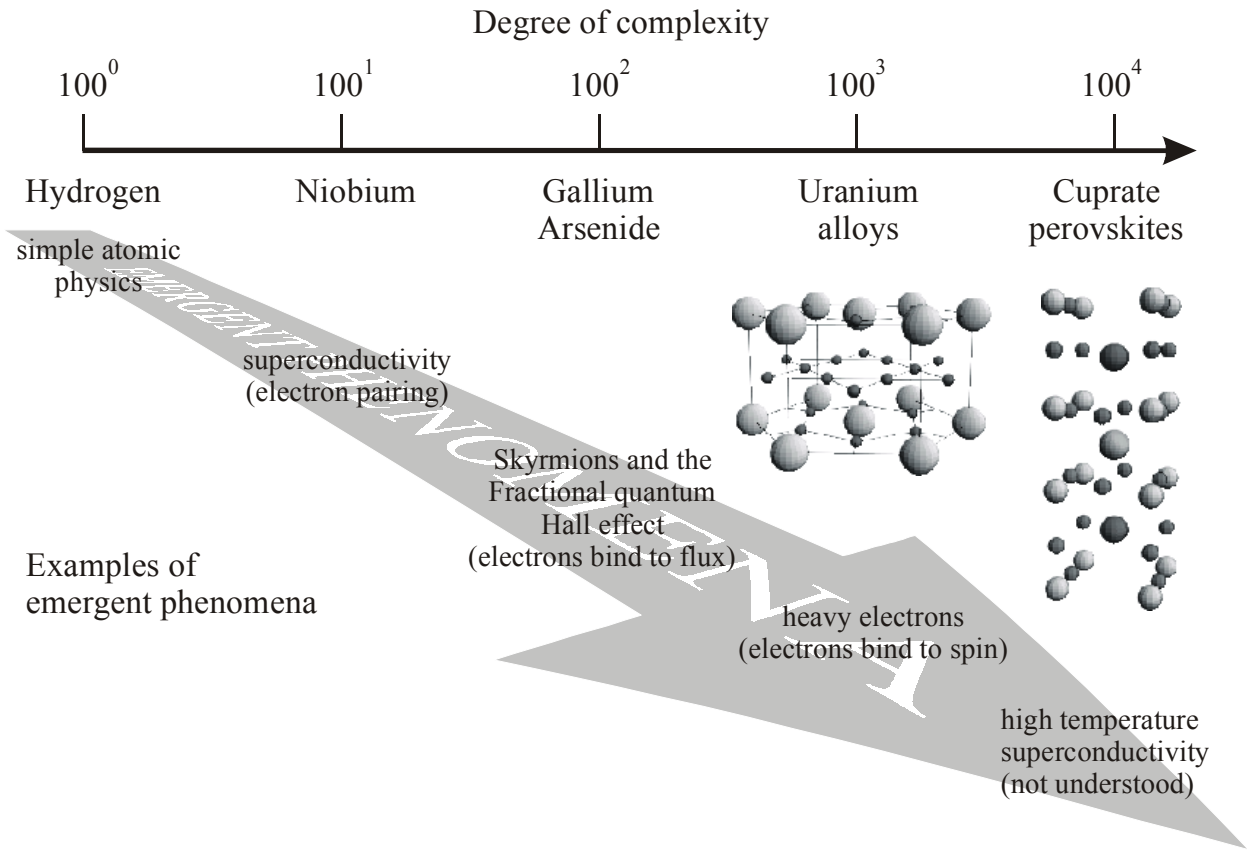

Figure 2: Condensed matter physics can be viewed as the exploration of quantum complexity. Every new element we introduce into a material's structure multiplies by 100 (roughly the number of elements) the number of compounds and hence the complexity. With every new level of complexity we see new properties appearing and their explanations can often require a very different view point from that suggested by the basic underlying ingredients of the electron and its quantum mechanical interactions.

Understanding how new physical phenomena emerge from complex systems often requires radically new concepts. One dramatic realization of this forms the subject of this article. Here, we focus on metallic solids where the interactions between the electrons can lead to strong correlations in their motion. As a consequence, we find exotic quantum phases of matter where *new* types of particles (such as fractionally charged objects) provide a much better basis for their description than that of the electron. We start by reviewing some of the basic concepts behind our current understanding. We will then describe recent discoveries that appear to demand a whole new conceptual framework for the theory of metals. Not only do these discoveries pose a challenge at a theoretical level, they also point to opportunities on a technological front.

The electron fluid: a quantum liquid

An important cornerstone in condensed matter physics is the theory of metals. Metals, such as copper, conduct electricity. In other words, electrical currents flow from one end of a piece of

metal to the other when, for instance, a nine-volt battery is connected across it. The amount of current that flows depends on the material in question. The current would be large in "good" metals (low resistance) but small in "bad" metals (high resistance). To understand the mechanisms that cause resistance, we need to have a microscopic model of a metal. What are these mobile charges? How do they flow? The search for answers to these questions takes us into the realm of quantum mechanics.

The electron: charge and spin

The discovery of the electron marks the beginning of modern physics. In trying to understand its behaviour, we discovered quantum mechanics. The electron is a deceptively simple object. It carries an electric charge e and is therefore responsible for all things electrical. The unit of charge, e, is fundamental in the sense that the electron cannot be split and that there are no other particles have been isolated with a smaller charge. However, this does not mean that there are no physical objects with a fraction of an electronic charge. Fractional charge does appear in the context of the fractional quantum Hall effect (see later).

The electron also possesses a magnetic moment or *spin*. In other words, it behaves in many ways like a tiny bar magnet. For instance, when placed between the poles of a horseshoe magnet, it will try to align its moment in the same direction as the magnetic field generated by the magnet, that is, towards the south pole. The important difference lies in the quantum nature of the spin—there are only two independent spin states corresponding to opposite spin directions (frequently referred to as "up" and "down"). In a magnetic material, such as iron as found in a bar magnet, these spins align parallel with each other, forming a large total magnetic moment. This is a ferromagnet. There are also antiferromagnets where neighbouring spins align in opposite directions. One of these materials gives rise to the class of high-temperature superconductors (which will also be discussed later).

Shortly after the discovery of the electron, the first progress in understanding the difference between metals and insulators was made. In an insulator like rubber, the electrons are tied down to specific atoms or and are not free to move around. Metals, on the other hand, should be viewed as a collection of atoms each of which has given up, on average, one to two electrons to be shared among the other atoms. These electrons can conduct electricity because they are now free from their parent atoms and are completely mobile. This "sea" of free electrons is quite different from any classical gas of particles like the helium gas used to fill balloons. Helium atoms at room temperature move much like randomly colliding snooker balls governed by Newton's laws of motion. However, we know from atomic physics that electron motion cannot be fully understood through these same laws of classical mechanics, for otherwise negatively-charged electrons would quickly fall into the positively-charged nucleus. In order to understand electrical conduction in metals, we have to take seriously the fact that these electrons form a *quantum* fluid.

Electrons in quantum mechanics are said to live in "quantum states" (see Figure 3.) A central principle governing the physics of electrons is the Pauli exclusion principle — two electrons cannot be in the same quantum state. Electrons prefer to arrange themselves so that their total energy is as low as possible. However, if one electron is already occupying a low-energy state, another one cannot adopt the same state and is forced to occupy one with higher energy. Some of these electrons end up have very high energies indeed (see Figure 3.) In a typical metal such as copper, an electron has an average speed of 10^5 metres per second. In fact, this is fast enough for

an electron to be shot into space escaping the gravitational pull of the Earth. The reason that this does not happen is that the negatively charged electrons are held back by the strong electrostatic attraction from their parent atoms (which are now positively charged "ions" after losing an electron to the conduction sea.)

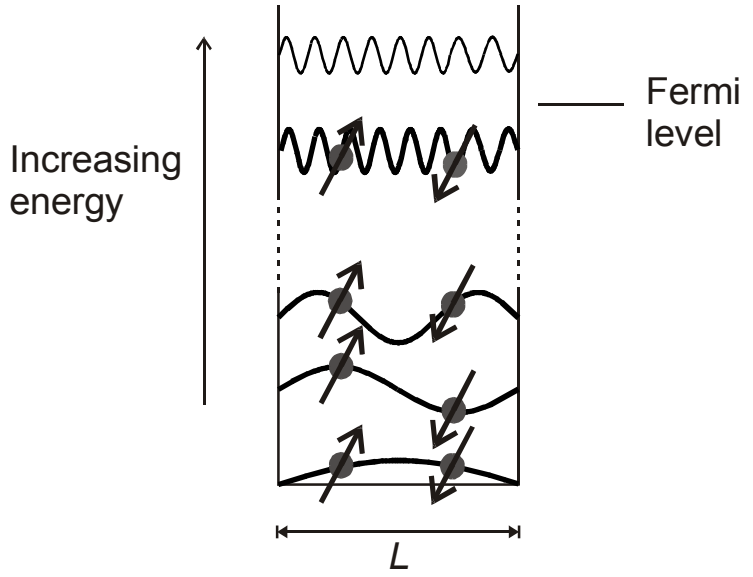

Figure 3: Particles in a Box.
In quantum theory, particles like electrons can also be described as waves trapped by the sides of the material in which they reside. These are like the standing waves that can be formed by jiggling a rope tied to a tree or the vibrations on a violin string. The momentum of the particle, *p,* is related to the wavelength, λ, by de Broglie's relation: $p=h/\lambda$ (where *h* is Planck's constant). Only specific wavelengths are allowed if we are to fit the standing waves between the end walls of the solid: $\lambda=2L/N$, where *N* is a whole number and *L* is the separation of the walls. We say that this *quantizes* the momentum: $p=Nh/2L$. The system favours the configuration of minimum total energy, the "ground state". For one electron, this is the state with lowest momentum. For many electrons, we have to take into account the Pauli exclusion principle which states that only two electrons (with opposite spins) can be in the same quantum state. If we assume that the electrons in a metal do not interact with each other, then it is easy to determine the ground-state configuration. Working upwards in energy, we fill up the available standing-wave states one by one with two electrons. To accommodate all the electrons (typically 10^{23}), we will reach waves of very short wavelengths, *i.e.* high momenta and therefore high energies. In a real material, the wave with the highest energy will have a wavelength as short as inter-atomic distances. This top energy level is called the "Fermi level" and this configuration for the electrons is called the "Fermi sea".

We can now build a physical picture for electrical conduction in metals. These fast electrons are accelerated by the electric field that directs from one pole of the battery to the other. This is analogous to a ball rolling down a slope, except that the slope is of electrical origin. An electron can bump into impurities and the positively charged ions in the metal. These can deflect the particle away from the direction it is supposed to be following under the influence of the electric field. It may even be bounced backwards. This is now analogous to a ball falling through a pinball machine. The electrons are hindered in their forward motion by these obstacles so that they do not accelerate indefinitely. They settle down to a steady speed and so we observe a steady

current for a given applied voltage. This scattering is the origin of electrical resistance (which is voltage divided by current — Ohm's law.)

What about electrons getting in each other's way? After all, they repel each other because they have the same charge, and so they must scatter each other as well when their paths come close together. It turns out that this mechanism is relatively unimportant. Part of the reason is that, when two electron scatter off each other, one typically becomes faster and the other slower. As we have already emphasised, other electrons already occupy the states with lower energy (and speed). The Pauli exclusion principle therefore forbids these scattering events!

Nevertheless, the fact that an electron has to swim through a sea of other electrons does have consequences on the properties of the electron. The electrons appear to become heavier in the sense that they are harder to accelerate. We say that the charge carriers appear to be electrons with an "effective mass" that is larger than the free electron mass. This can be a significant effect in cases where electron-electron interactions are strong—the effective mass can be hundreds of times larger than expected. An intuitive picture for this effect is that the electron must push aside other electrons as it moves. We can say that the electron carries with it a positive charge cloud (that is, the space it has created by pushing aside negatively charged electrons.) The Russian physicist, Landau, has coined the term *quasi-particle* for this electron-like object, to distinguish it from the free electron.

This description of electrical resistance essentially assumes that the system behaves like a collection of electron-like objects that do not see each other. This concept is central to our understanding of everyday metals that we find in copper wires and semiconductor chips. The idea is known as Fermi liquid theory and was formulated by Landau in the 1950's. The success of Fermi liquid theory is remarkable, given that electrostatics gives strong repulsive forces between electrons. However, these electrons are travelling at high speed. Their paths only come close to each other fleetingly, and so their motion is not drastically affected by these brief encounters. We have seen that the high electron velocities arise from the need to accommodate the Pauli exclusion principle. The electron fluid is therefore a system ruled by the laws of quantum mechanics.

The concept of the quasi-particle lies at the heart of Fermi liquid theory. It is important to note that they are not just theoretical constructs — they are the "elementary particles" of condensed matter physics since they determine all the physical properties of the system. To borrow a phrase from high-energy physics, Fermi liquid theory has become the "standard model" of metals. It explains why a wide range of compounds made up of elements from different parts of the periodic table might behave in a similar way. However, it is by no means a "grand unified theory". There are many exceptions to the rule that point us towards novel phenomena. Indeed, they challenge us to find a new conceptual framework for the theory of metals. As hinted in our introduction, we are compelled to look for new "elementary particles" in the quantum fluid, which are radically different from the free electron.

Beyond Fermi liquids

Some materials behave like an ordinary metal at room temperature but they show completely different behaviour at low temperatures. We say that the system has a transition from a metallic phase to a non-metallic phase. This transition occurs at a very precise temperature, in the same way that water turns to ice at precisely zero degrees Celsius (O°C). This is a clear case of the breakdown of Fermi liquid theory.

To give a concrete example, some metals become superconducting at low temperatures: below -266°C for lead and below -250°C for niobium germanium (Nb_3Ge). Above the transition temperature, the material is an ordinary metal. Below it, there is no sign of electrical resistance at all. The material also becomes a strong diamagnet—expelling magnetic flux from its interior (the Meissner effect). When magnetic flux is forced into the material with a strong magnetic field, it enters as thin lines, each carrying a *quantized* amount of flux, $h/2e$.

What has happened? Clearly, the system has undergone some radical re-organization. In this case, the electrons (or, more precisely, the electron-like quasi-particles) have paired up. The origin of the electron pairing was explained by Bardeen, Cooper and Schrieffer in 1957 as the result of an attractive interaction mediated by the sound waves in the solid. It is one of the most successful theories in condensed matter physics.

The new entities (Cooper pairs) belong to a different class of particles from electrons. Like photons, which are particles of light, they are not subject to the Pauli exclusion principle. Instead, they do exactly the opposite—they *prefer* to occupy the same quantum state. They behave as if they are a single collective entity. If you want to deflect them, you have to deflect them *all* at the same time! This cannot be done by single point-like impurities (as is the case of ordinary metals) and so this quantum fluid does not display any electrical resistance. There is in fact an analogous phenomenon in optics: the laser where photons act as a single entity.

In this example, the Fermi liquid breaks down at low temperatures, giving way to a new state of matter. Other examples include ferromagnetism (for iron, below 773°C) where the electrons align their magnetic directions spontaneously without the need for an external magnetic field. However, much of the recent interest is in materials where we see a metallic state with a low (but not zero) resistance, but very different from the one that Landau envisaged. We have a growing number of examples of metals that do not appear to be describable using electrons (or electron-like quasi-particles.) These metals pose a great challenge to our conventional views of metallic, superconducting and magnetic behaviour. Among them, the materials with the greatest technological importance are arguably the cuprate superconductors where the metallic state (which exists at temperatures above the superconducting transition temperature) has been the subject of controversy ever since its discovery in 1987.

The mystery of the cuprate superconductors

The cuprates encompass around thirty distinct crystalline structures and contain upwards of three different elements. It is perhaps not surprising that new phenomena emerge in these compounds that are not found in simple solids like simple copper. Whereas for other metals the onset of superconductivity typically occurs at tens of degrees above absolute zero (-273°C), the cuprates have transition temperatures around 100 degrees above absolute zero (for example, -183°C for yttrium barium copper oxide, $YBa_2Cu_3O_7$). The practical significance is that this is above the boiling point of nitrogen (-196°C) and so this inexpensive liquid can be used as a refrigerant instead of liquid helium for other materials. Moreover, the existence of the cuprates raises hopes for the discovery of room-temperature superconductors. This will remain a major driving force behind superconductivity research in physics, chemistry and material science.

In spite of a decade of intensive research, we still do not have a good theoretical understanding of the cuprates. This is because there is an interplay of interactions that influence the physics of the electron system. However, we are coming to the consensus that the answer must lie in a common feature of all the cuprates—the copper and oxygen atoms are arranged in

layers. The other atoms act as a sort of scaffolding keeping these layers apart (see Figure 4). In each of these layers, the copper atoms are arranged on the corners of a square lattice. The oxygen atoms sit between neighbouring copper sites. In the pristine state of, say, lanthanum cuprate (La_2CuO_4), each copper atom donates one electron as a charge carrier. These electrons prefer to run around within each layer, only crossing from one layer to the next very occasionally. A first guess for a model describing this system would be to consider electrons hopping from site to site on a square lattice. The two-dimensional nature of this motion is believed to play an important part in the behaviour of this system. This has, in part, prompted a search for unusual materials in low dimensions.

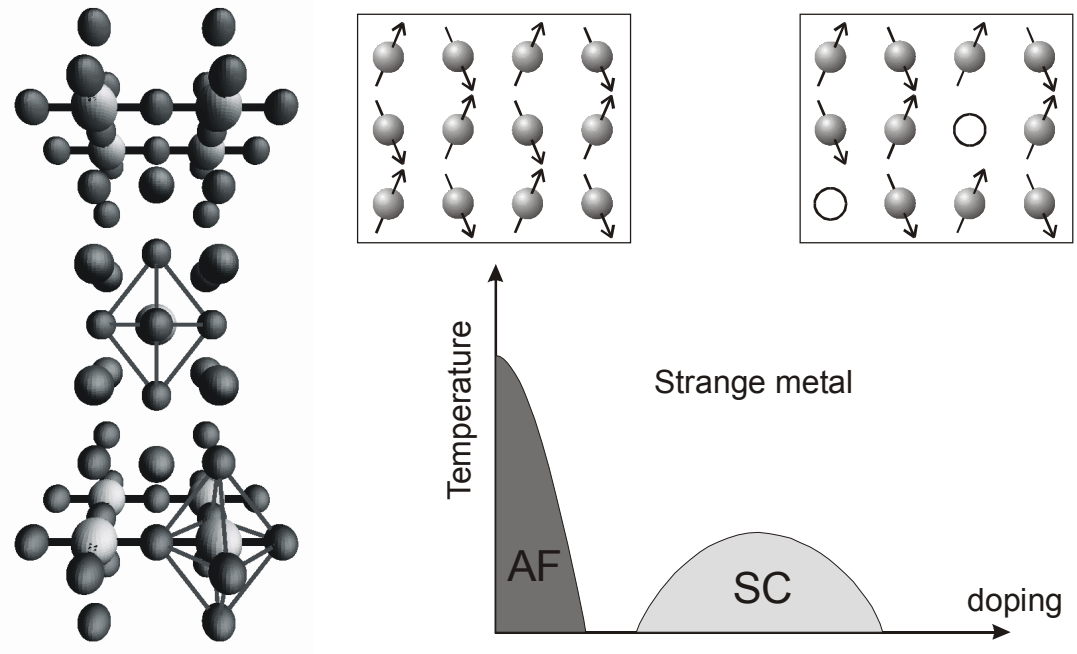

Figure 4: The high temperature cuprate superconductors exhibit a wide range of properties outside that expected for conventional metallic behaviour. A typical material (lanthanum cuprate is illustrated here) has layers of copper-oxide. In their pristine state, these layers are usually insulators with an antiferromagnetic arrangement of spins (AF). They are made superconducting by removing electrons from the layers – a process known as doping. This makes the electrons mobile, destroys the magnetism and creates a superconducting material (SC). The metallic state from which superconductivity emerges is so unusual that it is often simply called the 'strange metal phase'.

Although a non-interacting electron theory would predict it be a metal, La_2CuO_4 is an insulator. The reason is that the repulsion between electrons is important here, and this has caused an electronic traffic jam. We have one electron per copper site, and an electron cannot hop to a neighbouring site because of the repulsion from the electron that is already there. This kind of insulating behaviour is called a Mott insulator. Mott insulators are expected to be antiferromagnets—the magnetic moment on copper alternates in direction as one moves from one site to its neighbour. This is observed in all the cuprates.

To make these insulators conduct electricity, we have to relieve the traffic jam. We can do this by changing the chemical composition of the compound so that electrons are transferred from the copper-oxide layers to the atoms residing between the layers. (For La_2CuO_4, we can introduce

strontium, which substitutes for the lanthanum atoms.) In this way, electron vacancies (holes) can be created in the layers. The electrons can now move by hopping into the vacancies. Another way of looking at this is to watch the vacancies being shunted around the lattice by the motion of the electrons. The holes have a positive charge relative to the original insulating compound and so we can attribute the conductivity to mobile, positively charged holes.

As we increase the number of electron vacancies, antiferromagnetism soon disappears and the material becomes a metal. This metal becomes superconducting at low temperatures (Figure 4). The transition temperature reaches a maximum of about -170°C when the lattice contains 15-25% vacancies. (The record is -120°C for a similar compound involving unfortunately highly toxic mercury.) One might therefore argue that antiferromagnetism and superconductivity compete here. On the other hand, the superconductivity is intimately related to the proximity of the conductor to a magnetic state—the superconductivity disappears at higher vacancy concentration when we are far from the original antiferromagnetic insulator.

Leaving aside the superconducting state, the metallic state of the cuprates is itself unusual. One of the earliest observed anomalies is the linear proportionality between the resistance for currents in the copper-oxygen planes and the temperature. This is a robust phenomenon that occurs for nearly all the cuprates when the doping is optimal, and it holds over a wide range of temperatures, from the superconducting transition to 700°C. Moreover, the origin of this resistance apparently involves only the electrons. This is surprising because no known theory predicts this behaviour from electronic scattering alone. In fact, Fermi liquid theory expects the resistance to be proportional to the *square* of the temperature, through the scattering of one quasi-particle by another.

We can also measure the scattering rates of quasi-particles by observing the sideways current generated by the Lorentz force on the electrons due to a magnetic field (the Hall effect). Curiously, the resistance for these currents *does* give a scattering rate proportional to the square of the temperature. How can a single quasi-particle have two radically different scattering rates? The answers to this question and many other peculiarities of the cuprates remain highly controversial. Even the co-authors here have different proposals!

As hinted above, a theoretical model would have to give a quantitative description of how the mobility of the electronic charge and the antiferromagnetic correlations of the electronic magnetic moments are intertwined together. Unfortunately, even the simplest theoretical models that include these effects are mathematically intractable beyond one dimension. At present, it is also unclear how such a model would give rise to a mechanism for superconductivity.

Nevertheless, many theorists are coming to the view that the experimental anomalies point to a total breakdown of the Fermi liquid in the metallic state of the cuprates. In other words, the fundamentally current-carrying object might be radically different from the electron or any electron-like quasi-particle. The ingredient essential to this system appears to be the presence of some magnetic correlations.

One fruitful direction of research is to broaden our perspective to see if a larger range of magnetic materials exhibit similar non-Fermi liquid behaviour. In fact, the study of the magnetic metals and insulators pre-dates the discovery of the cuprate superconductors. These compounds exhibit a wide range of behaviour, such as ferromagnetism, antiferromagnetism and superconductivity. For instance, cerium palladium silicon ($CePd_2Si_2$) is an antiferromagnetic metal at atmospheric pressure. The antiferromagnetism can however be destroyed at high pressures, and the material becomes superconducting on the disappearance of magnetic order (see Figure 5). The superconducting critical temperature is very low (0.4K), but the apparent

competition between antiferromagnetism and superconductivity is qualitatively similar to the cuprates. This compound might shed some light on how the Fermi liquid state evolves towards a transition where a new quantum state of matter emerges. In particular, it belongs to a class of materials known as "heavy-fermion compounds" where a Fermi-liquid interpretation would deduce a quasi-particle which has a mass hundreds, or even thousands of times greater than the electron mass. We can regard this as a signal that Fermi liquid theory has been stretched to the limits of its applicability in these materials. The elucidation of the actual breakdown of the Fermi liquid state remains a challenge for the future.

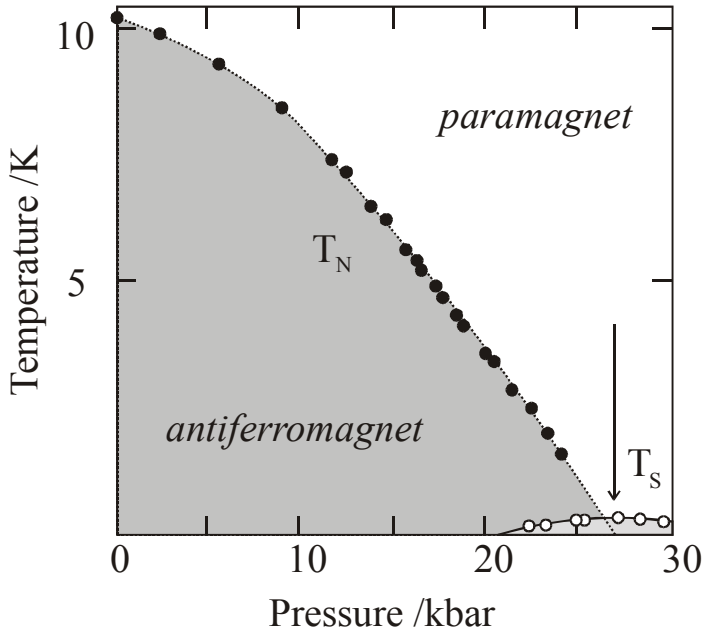

Figure 5: The interplay between magnetism, superconductivity and metallic behaviour can be studied by applying pressure as well as by doping as in the case of the cuprates. This has the advantage of minimizing the role of disorder. In this material (cerium palladium silicon, $CePd_2Si_2$, studied by Julian and co-workers in Cambridge), the magnetism can be squeezed out of the metal at high pressure (28,000 atmospheres). Near the point where the magnetism is lost, electrons interact strongly with the incipient magnetic order, and the whole notion of an electron seems to break down. The theory describing what happens here does not conclusively match with the known experimental results. Tantalizingly, at this very point on the phase diagram (indicated by the arrow), superconductivity appears.

Novel Particles?

If we do not have a Fermi liquid, we need to look for a new way of characterizing the system. Does this mean that the system is now so complex that it defies description by any simple physical picture? Our hope is that even complex systems have organized ground states, and that these quantum states of matter can be characterized by novel quasi-particles. We describe below a few examples where non-electron-like quasi-particles are known to exist. Notice that, in addition to strong interactions, all these systems involve physics in an effectively reduced number of spatial dimensions.

Spin-charge separation

One conjecture, put forward by Philip Anderson, for understanding the metallic state of the cuprates is that charge and spin (magnetic moment) have somehow become de-coupled in this system. Whereas charge and spin were tied together on the same object (the electron or the electron-like quasi-particle) in Fermi liquids, Anderson proposed that there are two separate quasi-particles, one for charge ("holon") and one for spin ("spinon"), and that an added electron "falls apart" into these two objects! More precisely, the electrons in the system have re-organized themselves so that charge and spin disturbances can become widely separated in space.

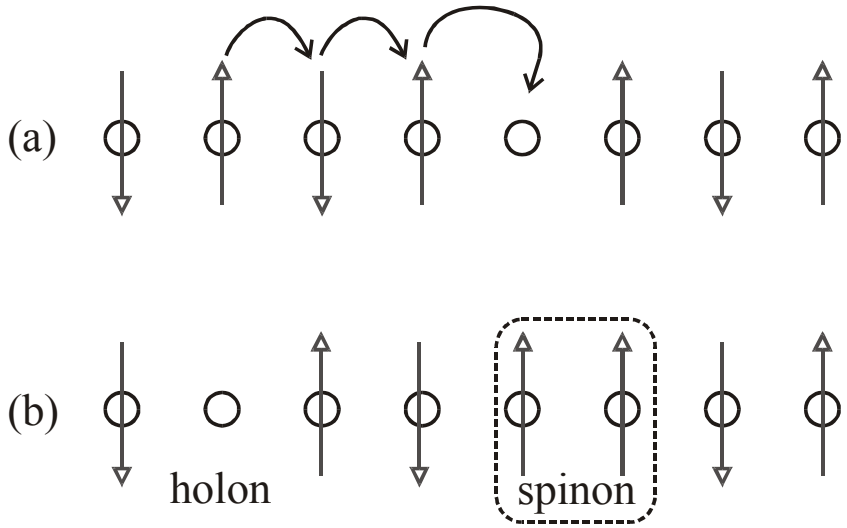

Figure 6: In one dimension, the electron can appear to fall apart into separate entities carrying the electrons' spin (the spinon) and charge (the holon). (a) Removing electrons from an ordered antiferromagnetic state, we remove both some spin and some charge. As electrons hop through the lattice we reach a state where (b) the disruption in the spin ordering has moved away from the disruption in the charge ordering. This gives a crude picture of spin-charge separation. Philip Anderson has made the bold suggestion that this physics of one dimension may apply also to the high-temperature cuprates where the copper-oxide planes are effectively two-dimensional. Considerable theoretical and experimental effort is being invested in exploring this possibility.

There is evidence of this spin-charge separation in the one-dimensional analogue of the cuprates. Consider a chain with one electron per site on the chain. As in the cuprates, electrostatic repulsion makes this an antiferromagnetic Mott insulator. So, the spin direction of each electron is antiparallel to its neighbours. Suppose we remove a spin-down electron from this chain (Figure 6a). Now, we can move the electron on the left of the vacancy onto this site, *i.e.* the charge disturbance (holon) moves to the left. Note that we have created a defect in the antiferromagnetic spin alignment—there are two neighbouring electrons with *parallel* spins. This is a spin disturbance (spinon) relative to the original state. If we iterate this procedure with the new vacancy, we can move the charge disturbance (holon) far away from the defect in the spin arrangement (spinon) (see Figure 6b).

This illustration of "spin-charge separation" can be formalized into a quantitative theory in one dimension. This has led to the search for quasi-one-dimensional compounds that might exhibit this behaviour. Strontium copper oxide ($SrCuO_2$) is a promising candidate with copper-

oxygen chains (instead of planes). It is a Mott insulator in its pristine state and an experiment like that suggested in Figure 6 can be done by ejecting electrons with light. When this is done there is evidence for seeing two moving particles instead of the single empty site left behind. Although the two-dimensional problem relevant to the cuprate superconductors has no mathematical solution so far, our progress with their one-dimensional relatives is encouraging.

Fractional charges

Other condensed matter systems also exhibit novel coupling between spin and charge where, in contrast to the electron, different proportions of electric charge and magnetism are bound together in the elementary excitations. An example can be found in semiconductors in high magnetic fields.

Semiconductor devices, such as the field-effect transistor, make use of electrons confined at the interface between two semiconductors. This is an effectively two-dimensional sheet so that reduced spatial dimension again comes into play. Suppose a current, I, is flowing down the sheet due to an applied voltage from a battery. In addition, the sheet is placed between the poles of a magnet so that the magnetic field is perpendicular to the plane of the electron sheet. The flow of electrons bends round due to the Lorentz force from the magnetic field. This generates a sideways voltage, $V_{H,}$ *perpendicular* to the current flow). The Hall voltage and the current are proportional to each other, and so we can define a Hall resistance: $R_H = V_H / I$.

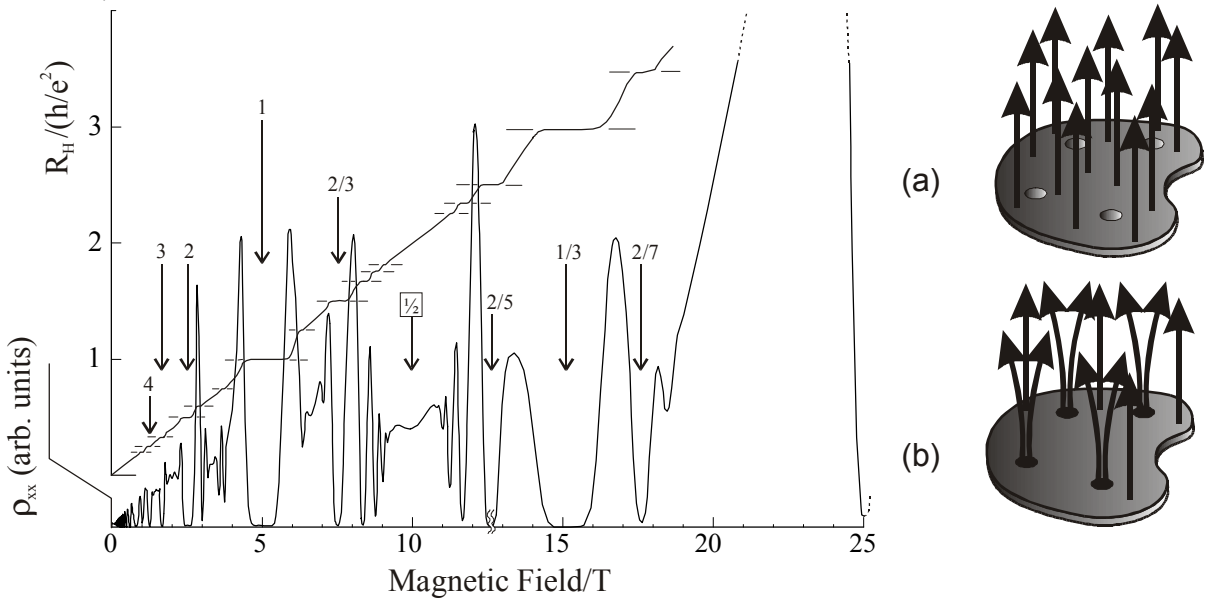

Figure 7: The fractional quantum Hall effect is an example of how fractionally charge objects can appear in a condensed matter system. Electrons confined in two dimensions show unusual stability when the number of electrons per magnetic flux (the *filling*) is a whole number. This "integer quantum Hall effect" can be seen by dips in the resistivity, ρ_{xx}, or plateaux in the Hall resistivity, R_H. The unexpected observation of dips (and corresponding plateaux) at fractional fillings was first understood by Bob Laughlin who showed how fractionally charged objects must be responsible. Jainendra Jain showed how the fractional effect is related to the integer effect, by (a) electrons binding to magnetic flux to form (b) composite particles, leaving one free magnetic flux per particle. In other words, the fractional effect is the integer effect for the composite particles.

For small magnetic fields, the Hall resistance increases smoothly with the magnetic field. At strong fields, its evolution develops steps and plateaux. In fact, it becomes *quantized* at particular fractions of h/e^2: *i.e.* $R_H = h/\nu e^2$ for ν=1,2,3,… This is the "integer quantum Hall effect" ("integer" because ν is a whole number.) This quantization is so accurate and robust to extraneous effects such as disorder that it has become the metrological standard for electrical resistance.

For very clean systems and even lower temperatures, we observe a "fractional quantum Hall effect" as well with additional plateaux of the Hall resistance corresponding to fractional values of ν, such as ν =2/3,2/5,1/3 (see Figure 7). The discovery and exposition of the fractional effect have won Laughlin, Störmer and Tsui the 1998 Nobel Prize. The most spectacular property of this system is that charge excitations come in fractional units of the elementary charge. When the Hall resistance of the system is at a plateau value of $3h/e^2$, the quasi-particle appears to carry charge $e/3$!

Have we split the electron? No. This fractional charge emerges from this electron quantum fluid through the co-operation of many electrons. In fact, all the electrons have conspired to organize themselves into a novel state. Here, the electronic motion is highly correlated with electrons circulating around each other in small groups of three, continuously changing from one group to the next. From this dance of the electrons emerges the physics of fractional charge.

Interestingly, the idea of magnetic flux quanta (in units of h/e) from superconductivity theory enters this story as well. For the c=1/3 plateau to occur, the magnetic flux density has to be near 3 flux quanta per electron. It turns out that an elementary excitation of this system is the introduction of a unit flux quantum which, from the flux-per-electron counting, corresponds only to one third of the electronic charge. Again, it should be emphasized that these fractional charges are not theoretical constructs. Indeed, they have been observed directly in recent experiments.

Skyrmions

The integer quantum Hall state also has interesting quasi-particles. In the previous section, we have assumed that all the electron spins are aligned with the external magnetic field. This is the Zeeman effect. Recent experiments have shown that applying pressure to the solid can weaken the Zeeman effect, and so spin reversals are now possible.

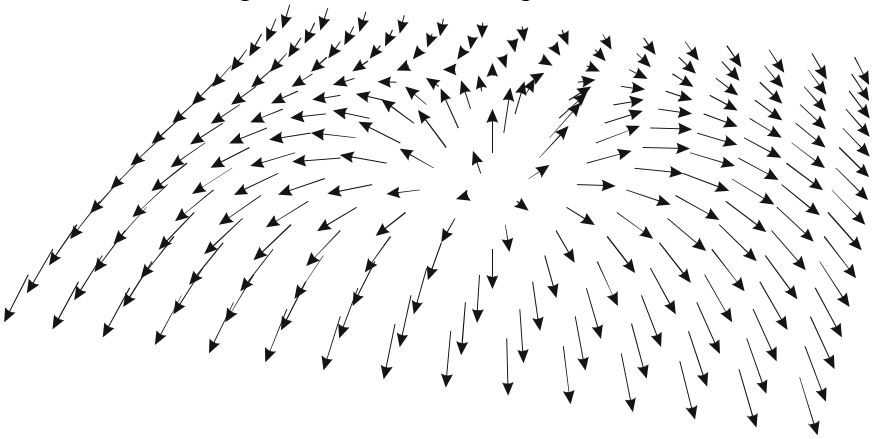

Figure 8: A skyrmion is a localized spin arrangement (known as a texture). Close to the center of the skyrmion the electron spins align in one direction but far away this direction is reversed. This spin particle has been discovered in quantum Hall systems.

It turns out that, even in the absence of an aligning field, the electrons in the quantum Hall regime prefer to align their spins parallel to each other. In other words, it is an intrinsic ferromagnet, like an iron bar. Consider adding an electron to this ferromagnet. In the integer Hall state, all the quantum states of one spin state have been occupied, and the Pauli exclusion principle forces the additional electron to occupy a state with its spin antiparallel to its neighbours. The inherent ferromagnetism of this electron gas does not favour this misalignment and so the neighbouring electrons attempt to rotate their spins to smooth out the disturbance. This may involve up to 20 spins around the new electron (Figure 8). We see that adding one electron to the quantum Hall system results in the addition of one unit of charge *e* but *many* units of spin. This large-spin single-charge quasi-particle is called a *skyrmion*, named after Skyrme who originally discussed such objects in the context of topological field theories in high-energy physics. This skyrmion has finally been observed in the completely different arena of condensed matter physics.

Speculations

One of the attractions of condensed matter physics is that theoretical progress has implications for applications as well. Our understanding of electrons in solids, based on Landau's quasi-particle picture, underpins the silicon-based semiconductor technology that has transformed our lives in this century.

The discovery of high-temperature superconductors in 1986 by Bednorz and Müller has already opened up new opportunities in superconducting technology, previously ruled out because of the expense and inconvenience of liquid-helium cryogenics. The absence of any electrical resistance allows the storage and transmission of electricity with no losses. The manufacture of superconducting cables is becoming a commercial possibility. Large magnets using superconducting coils are in use today in a wide range of applications, from particle accelerators to medical magnetic resonance imaging. The Meissner effect of magnetic flux expulsion can be also exploited to achieve magnetic levitation, leading to the prospect of friction-free trains with superconducting coils gliding over magnetic tracks.

On a smaller scale, superconductor electronics are also in sight. The cuprate superconductors are already in use in superconducting quantum interference devices (SQUID's) which can detect minute changes in magnetic fields. These devices have a wide range of applications, from medical instruments to non-invasive testing for fracture in aircraft wheels.

The interplay of magnetism and electrical conduction has also found recent application in magnetic storage devices such as computer hard disks. Here one uses materials whose electrical resistance is strongly affected by a magnetic field (showing “giant magneto-resistance"). There are probably many such possibilities using strongly correlated materials where, as we have seen, the competition between magnetism and electrical conduction can play an important role. It is, for example, interesting to note that manganate relatives of the cuprates have even higher magneto-resistance (so-called “colossal magneto-resistance").

Looking further into the future, one might anticipate that a deeper understanding of the new types of exotic excitation arising in strongly correlated materials may lead to technologies every bit as rich as the single electron-quasi-particle physics which dominates the semiconductor device industry at present. Indeed if technological dreams such as quantum computers are to be realized in practice, we will need to exploit the macroscopic consequences of quantum mechanics which routinely dominates the physics of materials of current theoretical interest.

Just as the turn of the last century saw the discovery of the electron and the key to understanding simple metals in terms of the electron quantum fluid, we are now seeing an increasing need for more exotic quantum fluids to account for the behaviour of more complex metallic states. Already we have seen materials where the immutable electron should actually be viewed as separating into its magnetic and charged components. Also, fractionally-charged objects have been identified in semiconductor devices and the dreams of particle theorists are being realized in condensed matter systems.

These exciting developments are fuelled by the ever-increasing complexity of structures that material scientists can produce. From high-temperature superconductivity to colossal magneto-resistance, we are continually surprised by the new phenomena that nature throws us as we explore more complex systems. The search for new concepts to understand these phenomena is a major challenge facing the condensed matter physicist at the start of this new century. Technological applications are surely only a matter of time.

Further Reading

Philip W. Anderson, *Condensed Matter — The Continuous Revolution*, Physics World, December 1995.
Piers Coleman, *Condensed Matter — Correlated Electron Systems*, Physics World, December 1995.
T. Maurice Rice, Physics World, December 1999 (and other articles in the issue).
Dung-Hai Lee and Shou-cheng Zhang, *Electrons in Flatland*, Scientific American, March 1996.

Biographical Information

Derek Lee is a condensed matter theorist. He is interested in quantum phases of matter whose existence depend on quantum correlations, interactions and disorder. These include disordered superfluids, dirty metals and the cuprate superconductors.

Born in Hong Kong, Derek Lee studied Natural Sciences at Cambridge where he graduated with first class honours in 1987 and obtained his PhD in 1990. His postdoctoral work on one-dimensional metals, electron localization and the gauge theory of unconventional metallic behaviour in high-temperature superconductors was carried out in Oxford and, as a Lindemann fellow and a NATO/EPSRC fellow, at the Massachusetts Institute of Technology, USA (1993-96). In 1997, he joined the Condensed Matter Theory Group at Imperial College, London, as a Royal Society University Research Fellow.

Andrew Schofield is a Royal Society University Research Fellow and proleptic lecturer at the University of Birmingham. His research interests include the theory of correlated quantum system and their manifestation in oxide materials such as high-temperature cuprate superconductors and also in heavy fermion metals and semiconductor structures.

Originally from Surrey, Andrew (aged 33) graduated from Cambridge with a first class BA and Mott prize in 1989 and a Ph.D. in 1993 where he used a gauge theory approach to study doped Mott insulators. He then worked at Rutgers University, USA, studying Kondo models and developing a phenomenological approach to transport in the cuprate metals. He returned to Cambridge as a Fellow of Gonville and Caius College in 1996 and moved to Birmingham in 1999.