FQHE: The Motion Picture

MSci Project 2004

Interacting electrons confined to two dimensions exhibit the fractional quantum Hall effect (FQHE) in high magnetic fields. The Hall resistance becomes quantized in rational fractions of h/e^2. This is an example of new order arising from simple interactions in a quantum system. The Coulomb interaction is very strong in these systems and so we cannot use simple plane waves to describe these electrons. Laughlin's trial wavefunction has been spectacularly successful in describing the quantum ground state of the system (Nobel Prize 1998). He has shown that the electrons form a quantum fluid with strongly correlated motion.

This project attempts to obtain a semi-classical description of this quantum fluid. A quantum wavefunction gives us the probabilities for different electron configurations. It also provides the probabilities for the distribution of currents. So, we can use the Laughlin's wavefunction to obtain a set of typical classical trajectories for the collection of electrons. This will help us visualise the quantum correlations implicit in the Laughlin wavefunction. The objective is to find such trajectories and analyse correlations in the electron motion. One end product of the project is an animation of the electron dynamics.

I. BASIC CONCEPTS

You have to understand some basic concepts about electrons in a magnetic field.

- **Quantum mechanics in a magnetic field**

  Any standard textbook on quantum mechanics will have a section about the following. Try Gasiorowicz.

  - *Basics*: Wavefunctions, current/velocity operator.
  - *Vector potential*: vector potential for a uniform magnetic field. current operatur in a magnetic field.
  - *Landau levels*: quantum mechanics of electrons in a magnetic field. This is an important ingredient of the quantum Hall effect

- **Quantum Hall effect**

  Most literature is aimed at postgraduate level. Try looking for a brief discussion in modern solid-state textbooks, such as Singleton and Burns For non-technical accounts, try:

  D-H Lee and S-C Zhang, *Electrons in Flatland*, Scientific American, March 1996.
  Nobel Lectures 1985 (integer quantum Hall effect), 1988 (fractional quantum Hall effect): http://www.nobel.se/physics/laureates/

Cambridge Semiconductor Physics website:
http://www.sp.phy.cam.ac.uk/SPWeb/research/QHE.html
http://www.sp.phy.cam.ac.uk/SPWeb/research/FQHE.html

A few useful books are:

C. Kittel, Introduction to Solid State Physics
G. Burns, Solid State Physics
J. Singleton, Band Theory and Electronic Properties of Solids