Quantum Theory of Matter

Overview Lecture

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Outline

1. **Course content**
   - Introduction
   - Superfluids
   - Superconductors
2. **Course structure**
   - Course Plan
   - Resources

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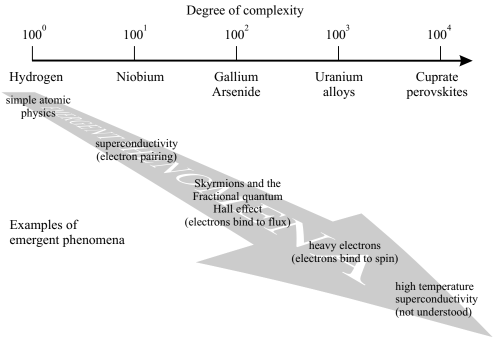
Complex Materials

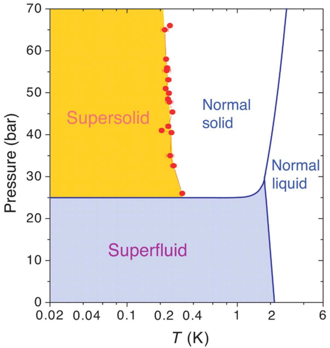


- Periodic table
  $\sim$ 100 elements
  $\longrightarrow$ binary compounds
  $\longrightarrow$ tertiary compounds
  ...
- Simple rules
  - quantum mechanics
  - Coulomb interactions

$\longrightarrow$ but complex cooperative phenomena emerge

The sum is more than its parts

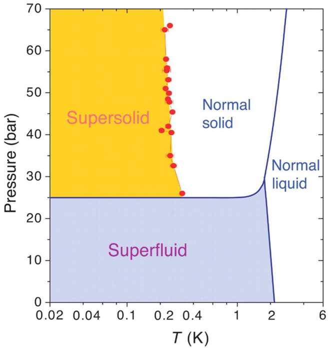
Superfluids



Kim & Chan, *Science* 2004

- Superfluids flow without viscosity
  - liquid helium ^{4}He (below 2K)
  - optically trapped alkali atoms (nK)
- Not dragged by container walls $\longrightarrow$ container rotates but superfluid stays still
- Bose-Einstein condensation

Superfluids



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- Bose-Einstein condensation
- supersolid ^{4}He ?!

A Question of Identity

- Two identical particles: complex wavefunction $\psi(\mathbf{r}_1, \mathbf{r}_2)$
  - Prob [one particle at $\mathbf{r}_1$ & another at $\mathbf{r}_2$] $\propto |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$
  - Indistinguishable particles $\Rightarrow |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2 = |\psi(\mathbf{r}_2, \mathbf{r}_1)|^2$

Only two possibilities in 3 dimensions:

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\psi(\mathbf{r}_2, \mathbf{r}_1)$$

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$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \pm\psi(\mathbf{r}_2, \mathbf{r}_1)$$

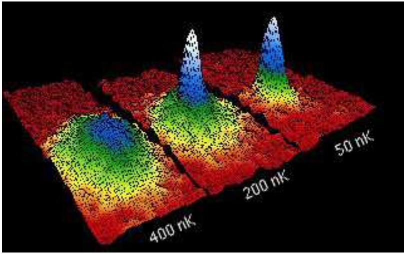
Fermions: $\psi(\mathbf{r}_1, \mathbf{r}_2) = -\psi(\mathbf{r}_2, \mathbf{r}_1)$

- electrons, protons, ^{3}He
- Pauli exclusion:
  no two particles in the same state

Bosons: $\psi(\mathbf{r}_1, \mathbf{r}_2) = +\psi(\mathbf{r}_2, \mathbf{r}_1)$

- ^{4}He, ^{87}Rb, photons
- no Pauli exclusion principle

Bose-Einstein Condensation



Bose-Einstein condensation

All particles in the one-particle eigenstate with lowest energy

Coherent quantum state → no scattering (“all in step”)
[coherent photons → lasers]

http://www.colorado.edu/physics/2000/bec/

^{4}He and ^{87}Rb atoms are electrically neutral.

Charged superfluids?

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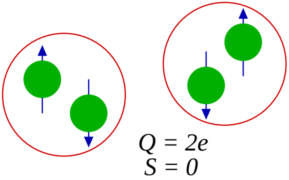
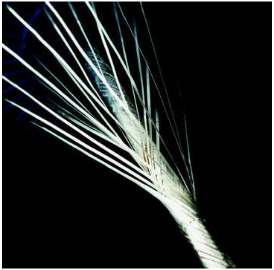
Condensation of electron pairs $\longrightarrow$
superfluidity of charge $2e$ bosons:
Cooper pairs



Superconductors

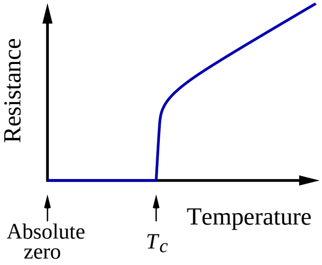


The Perfect Conductor

- Electrical current through a copper wire heats it up: $P_{\text{loss}} = I^2 R$
  - Good for heating
  - Waste of energy for light bulb, TV, computer...
  - A large fraction of electrical power is lost between the power generator and your home through heat loss

Superconductors have no electrical resistance when cooled to a low temperature.

Perfect electrical conduction



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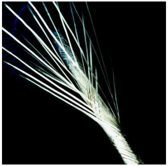
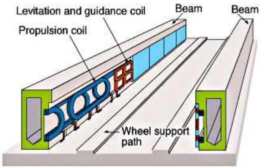


- Power applications
  - generation/transmission
- Powerful magnets
  - medical imaging (MRI)
  - maglev trains: 300 mph (Yamanashi test line)
  - particle accelerators

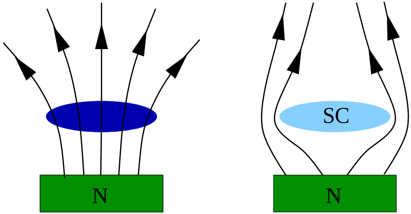


Meissner Effect

Perfect diamagnetism: superconductors expel magnetic fields

Field penetrates a normal metal, but not a superconductor.

Currents flow to cancel applied external field.



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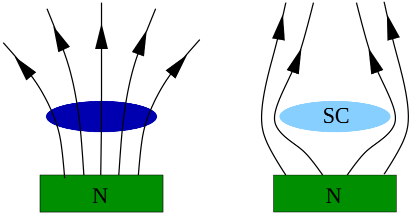
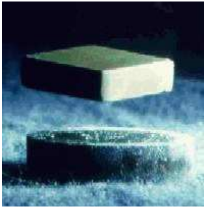


This generates an *upward* force on the superconductor.
(Analogy: garden hose) $\Rightarrow$ Superconducting Levitation

Superconducting Levitation



Magnet levitated by a superconductor

Superconducting Levitation

sumo levitation

Flux Quantisation

Aharonov-Bohm effect: (momentum $\mathbf{p} \rightarrow \mathbf{p} + q\mathbf{A}$)

Charges see magnetic flux through loop

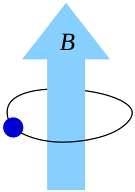
$$\text{Flux} = \Phi = \int \mathbf{B} \cdot d\mathbf{S} = \oint \mathbf{A} \cdot d\mathbf{r}$$

Cooper pair wavefunction acquires phase:

$$\psi \rightarrow \psi \, e^{2ie\Phi/\hbar}$$

Flux through superconductor is possible if

$$e^{2ie\Phi/\hbar} = e^{2\pi M i} = 1 \quad \Rightarrow \quad \Phi = M \frac{h}{2e}$$



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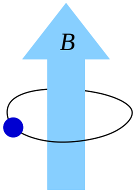
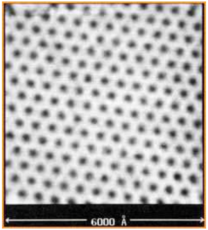




Magnetic field penetrates in discrete flux quanta of $h/2e$

STM image of vortex lattice

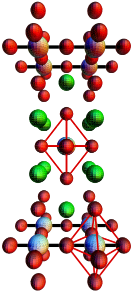
$NbSe_2$. Hess et al (1989).

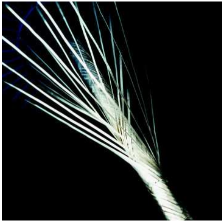
SQUIDs

Superconducting
QUantum
Interference
Device

- Current flows around coil to ensure quantised flux through centre
- Sensitivity to small local fields
  e.g. non-destructive testing of aircraft wheels

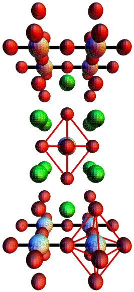
High-temperature Superconductors





- The cuprate family of superconductors, *e.g.* $\mathrm{La}_{2-x}\mathrm{Sr}_x\ \mathrm{CuO}_4$
- No electrical resistance below -120°C.
  This is warm! Liquid nitrogen boils at -196°C (77K).
- Cables and wires carry current with no heat loss
  $\Rightarrow$ power generation, medical imaging (magnets for MRI), . . .

A New Kind of Metal?



- $La_{2-x}Sr_xCuO_4$ ($T_c = 60$K) Bednorz & Müller 1986
  $YBa_2Cu_3O_{7-x}$ ($T_c = 90$K) Chu 1987
- Doping $x \simeq 10\%$ for superconductivity
- Common features
  - electrical conduction in CuO planes: "flatland"
  - parent compound ($x = 0$): not a metal but an *insulator*!
  - superconductivity in competition with *magnetism*

Does conventional theory of metals and superconductors still work?

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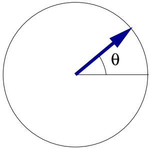
Phenomenology: Neutral Superfluids

- Bose condensation & superfluidity
  - ideal Bose gas: total Bose condensation
  - interacting Bose liquid
  - criterion for superfluidity
- Condensate wavefunction and phase

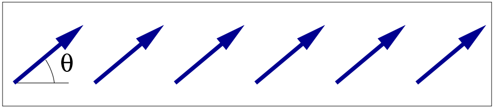
$$\psi_c = \sqrt{N} e^{i\theta}$$

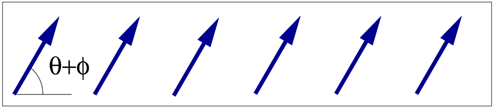


- Condensate phase twist = current $\quad \mathbf{J} \propto \nabla\theta$
  - Vortices: quantised angular momentum

Spontaneous Broken Symmetry

- Ferromagnet with spin rotation symmetry
  - global magnetisation can point in any direction
  - degenerate ground states can be generated by a global rotation of all the spins through an angle ϕ





- Magnet *spontaneously* chooses a magnetisation direction, breaking the spin rotation symmetry
- Superfluid: magnetisation $\rightarrow$ condensate phase

Phenomenology: Superconductors

- London theory
  - charged superfluid
  - condensate phase: coupling to vector potential

$$\nabla\theta \to \nabla\theta - 2e\mathbf{A}$$

  - London equation for screening of magnetic fields $\to$ Meissner effect
- Flux quantisation: flux quantum $h/2e$
- Josephson effect
  - phase coupling between superconductors
  - dc current without voltage

Many-body Theory

- From classical waves to quantum particles
  - sound waves $\longrightarrow$ phonons
  - electromagnetic waves $\longrightarrow$ photons
- Quantum field theory
  - particle creation and annihilation
  - elementary excitations in a correlated system
  - zero-point motion: vacuum fluctuations
- Applications
  - quantum crystals: zero-point vibrations
  - superfluids: zero-point phase fluctuations
  - conventional superconductors: Cooper pairing, BCS theory

Books

- J.F. Annett, *Superconductivity, Superfluids and Condensates* (Oxford Univ Press, 2004). Closest to course.
- D.R. Tilley & D.R. Tilley, *Superfluidity and Superconductivity* (Institute of Physics, 1990).
- P. Nozières & D. Pines, *The Theory of Quantum Liquids Vol 2* (Perseus 1999). Classic text aimed at postgraduates.
- P.G. de Gennes, *Superconductivity of Metals and Alloys* (Addison Wesley 1989). Classic postgraduate text.
- M. Tinkham, *Introduction to Superconductivity* (Dover 2004). Comprehensive postgraduate textbook.

Course material

- **WebCT** or:
  http://www.cmth.ph.ic.ac.uk/people/dkk.lee/teach/qtm/
  - course plan: aims & objectives
  - lecture notes (with errata)
  - problem sheets & solutions
  - recommended texts
- **Email**: dkk.lee@imperial.ac.uk
- **Office hours**:
  Monday 1-2pm, Thursday 4-5pm (Blackett 809)
- **Rapid feedback sessions**:
  Every other Friday 1pm, starting week 3.
  Demonstrator: Tanja Duric (tanja.duric@imperial.ac.uk)