Quantum Theory of Matter:

Superfluids & Superconductors

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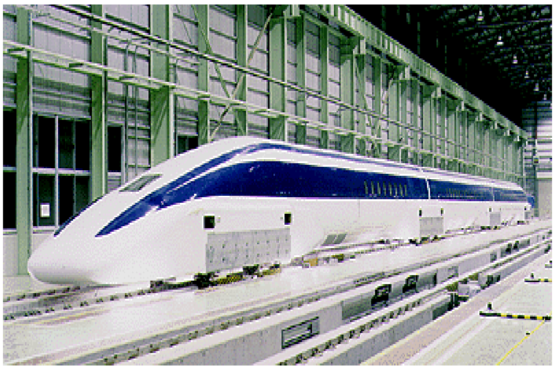
Level 4 course: PT4.5 (Theory Option)
http://www.cmth.ph.ic.ac.uk/people/dkk.lee/teach/qtm
Prerequisite: Advanced Classical Physics



Superconductors

zero resistance

flux expulsion

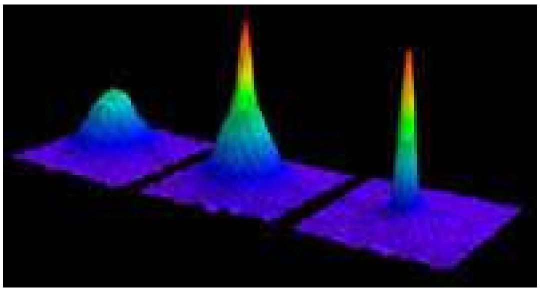
flux quantisation

Superfluids

atomic Bose condensates

liquid helium

quantised vortices



Concepts

spontaneous broken symmetry

macroscopic quantum coherence

quantum field theory in condensed matter physics

elementary excitations in strongly correlated systems

Introduction

Quantum mechanics was originally designed to describe the physics of one or a few atoms. We have since learnt that it can also dictate the physics of matter on a macroscopic scale. Superconductivity and superfluidity are perhaps the most dramatic manifestation of macroscopic quantum physics.

The electrical resistance of a metal decreases when it is cooled. For a superconductor, the resistance *vanishes* completely below a threshold temperature! The absence of dissipation has made superconductors useful in high-field magnets, *e.g.* in the LHC collider for high-energy physics or in a MRI machine in a hospital. A superconductor also expels magnetic fields, leading to applications such as magnetically levitated trains.

Superfluids, such as liquid helium and atomic Bose condensates, are the charge-neutral cousins of superconductors. Recent advances in quantum optics have led to an explosion of research activity on atomic condensates as a possible system for quantum information processing.

The key to the physics of these remarkable materials is the phenomenon of **spontaneous broken symmetry**. In this course, I will describe how this explains the macroscopic behaviour of superconductors and superfluids. This will be introduced in the framework of the Ginburg-Landau theory of **macroscopic phase coherence**. This theory gives a detailed description of the signatures of superconductivity: magnetic flux expulsion (Meissner effect) and flux lines carrying the magnetic flux quantum ($h/2e$).

I will also introduce the microscopic (Bardeen-Cooper-Schrieffer) theory of electron pairing which leads to superconductivity. This part of the course will contain a self-contained introduction to techniques in quantum field theory, such as second quantisation. Quantum field theory has been an important tool in condensed matter physics in the last 50 years. It is the natural way to analyse a system of many interacting particles whose dynamics are strongly correlated with each other. This will also provide us a glimpse of how ideas in physics cross disciplinary boundaries --- the mechanism of flux expulsion in superconductors is intimately related to (and pre-dates) the Higgs mechanism of mass generation in high-energy physics.

Throughout the course, systems of current research interest will be included not only to illustrate established theory but also to indicate where conventional understanding fails. At the end of the course, you will have a basic understanding of superfluids and superconductors. From a wider perspective, you will be exposed to key concepts and theoretical techniques in condensed matter physics.

Course Aims

- To illustrate the concept of **spontaneous broken symmetry** in quantum systems, with special focus on the theory of superfluids & superconductors
- To use systems of current research interest to illustrate the theoretical concepts
- To provide a microscopic theory of **neutral superfluids**, as applied to ***atomic Bose condensates*** and ***superfluid helium***
- To highlight the role of **phase coherence** in neutral superfluids
- To provide a quantum theory of the elementary excitations in a Bose fluid, emphasising its connection to the phenomenon of superfluidity
- To explain the Ginzburg-Landau phenomenological theory of phase coherence in **superconductors**, in order to explain the signatures of superconductivity:
    - Meissner effect (magnetic flux expulsion)
    - Josephson effect (current without voltage)
    - magnetic flux quantisation & the Abrikosov flux lattice
- To explain the microscopic origin of ***Cooper pairs*** in superconductors
- To describe the Bardeen-Cooper-Schrieffer microscopic theory of superconductivity
- To introduce techniques in **quantum field theory**:
    - second quantisation
    - elementary excitations
    - vacuum fluctuations
    - number-phase representation
    - Bogoliubov transformation

Prerequisites

Familiarity with the following concepts and techniques is highly recommended.

- Advanced Classical Physics (Level 3 option) is a pre-requisite to this course. Familiarity with the following topics will be useful:
  - Hamiltonian mechanics: normal modes.
  - Hamiltonian for motion in a magnetic field: momentum and use of vector potential.

- Concepts in Quantum Mechanics (Level 2 core)
  - Heisenberg uncertainty principle
  - Hamiltonians and Schrodinger's equation
  - Simple harmonic oscillator: energy levels, ladder operators. (We will revise this briefly at the beginning of the course.)
  - Quantum statistics: Bose-Einstein distribution, Fermi-Dirac distribution. See Statistical Physics (Level 2 core).

- Electrons in Solids (Level 2 core).
  - Free electron theory: electron Fermi surface, Fermi energy
  - Density of states

- Electromagnetism (Level 2 core)
  - Magnetic flux
  - Maxwell's equations: Faraday's law

- Mathematics (Level 2 core)
  - Fourier series and Fourier transforms
- Mathematical Methods (Level 2 option)
  - Calculus of variation: Euler-Lagrange equation

Course Format

The course consists **26 lectures** in the Spring Term, and a revision lecture at the start of the Summer Term. These lectures are delivered at the blackboard. They will concentrate on explaining key physical concepts.

A full set of **lecture notes** is provided. These expand on the content of the lectures. In particular, they will provide details of mathematical derivations. The current version of these notes is available online.

There are **problem sheets** to accompany the lecture notes. They aim to provide physical examples to illustrate key concepts from the lectures. There will also be exercises to reinforce learning of new techniques. Solutions are provided.

There will be **rapid feedback sessions** (one session every fortnight) where a demonstrator will be available to discuss and provide feedback on assigned problems from the problem sheets.

There are also 2 **office hours** per week for one-to-one discussion about the course.

Recommended Reading

Course texts

J. F. Annett, *Superconductivity, Superfluidity and Condensates,* (Oxford Univ Press 2004):

> The treatment of the subject in this book is closest to the course plan. (Central library is ordering copies.)

R.P. Feynman *et al*, *Feynman Lectures on Physics, vol III*, chapter 21 (Addison Wesley 1965):

> Nice introduction to phenomenological aspects of superconductivity.

Bose condensates

E.A. Cornell & C.E. Wieman, *The Bose-Einstein Condensate,* in *Scientific American* (March 1998).

Physics Nobel prize 2001: http://www.nobel.se/physics/laureates/2001/public.html

Superconductivity

Physics Nobel prize 2003: http://www.nobel.se/physics/laureates/2003/public.html

Quantum physics in condensed matter

D.K.K. Lee & A.J. Schofield, *Metals Without Electrons, in Visions of the Future: Physics & Electronics,* ed. J.M.T. Thompson (Cambridge Univ Press 2001)

> A review on state-of-the-art research in the physics of complex correlated quantum systems. Available at:
>
> http://www.cmth.ph.ic.ac.uk/people/dkk.lee/research/cupchapter.pdf