4. (i) The order parameter for a superconducting phase transition is a complex number

$$\Psi(\mathbf{r}) = |\Psi(\mathbf{r})| \exp(i\Theta(\mathbf{r})). \tag{4.1}$$

(a) What is the physical interpretation of $|\Psi(\mathbf{r})|^2$? [2 marks]

(b) Explain why one can assume that $|\Psi(\mathbf{r})|^2$ does not change significantly in space, that is, we can write $|\Psi(\mathbf{r})|^2 = |\Psi|^2$. [2 marks]

(ii) You may assume without proof the equation of continuity for probabilities

$$\mathbf{j}_{prob} = \frac{1}{2m}[\Psi^\star(\mathbf{r})(-i\hbar\nabla - q_s\mathbf{A})\Psi(\mathbf{r}) + \Psi(\mathbf{r})(i\hbar\nabla - q_s\mathbf{A})\Psi^\star(\mathbf{r})]. \tag{4.2}$$

(a) Identify clearly all the terms in the equation of continuity and explain why the right hand side is a real number. [3 marks]

(b) Derive the equation for the electrical current density

$$\mathbf{j} = \frac{q_s}{m}[\hbar\nabla\Theta(\mathbf{r}) - q_s\mathbf{A}]|\Psi|^2. \tag{4.3}$$

[3 marks]

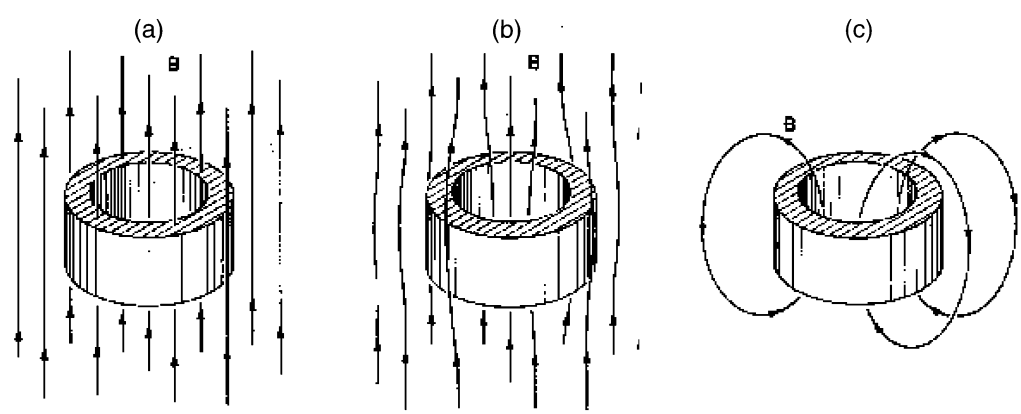
(c) What is the classical equivalent of Eq.(4.3)? [2 marks]

(iii) Consider a superconducting ring.

(a) Describe the phenomenon of flux quantisation. [2 marks]

(b) Show that Eq.(4.3) for the electrical current density implies flux quantisation. [2 marks]

(iv) Consider the diagrams below which display a superconducting ring in an external magnetic field at temperature (a) $T > T_c$ and (b) $T < T_c$. In (c), the external magnetic field has been switched off. Explain what happens when going from (a) to (b) and from (b) to (c). Indicate whether you will observe flux quantisation in the settings (a), (b) and (c). What is the source of the magnetic loops in diagram (c)? [4 marks]



[TOTAL 20 marks]