Core Course: What You Should Know

This set of notes gives an indication of the basic physics you are expected to know from the core course. It is not a comprehensive list of the syllabus. Hopefully, it gives you some perspective of the core course as a whole.

These notes are supposed to help you with your revision for the Comprehensive papers. (Some hints about exam technique are included.) However, it is important to remember that the Comprehensive questions are designed to test how you can apply basic concepts to solve physics problems. They are not designed to test how much detail you recall from individual courses.

1 General

1. **Dimensional analysis.** Check that your answers have the correct dimensions. This will pick up most errors in algebra.

2. **Algebra.** To avoid unnecessary mistakes, keep it neat and do not miss out too many steps. Try to reduce your formulae before evaluating them numerically.

3. **Diagram.** Draw a diagram to help visualise the problem. Define a suitable coordinate system if needed – this helps you get the correct signs.

4. **Vectors.** Don't mix vectors with scalars. Otherwise, you will lose information about the direction of a vector quantity (*e.g.*, momentum, electric field.) Make sure you know the direction of a cross product.

5. **Approximations.** Use Taylor expansions for small changes, *e.g.* $\sin x \simeq x, \cos x \simeq 1 - x^2/2, (1+x)^\alpha \simeq 1 + \alpha x, \ln(1+x) \simeq x$ for $x \ll 1$.

6. **Differential equations in physics.** Use of small volume elements in deriving a differential equation describing the physics of the problem. (*e.g.*, in mechanics, consider the forces applied on the element and the acceleration that arises from these applied forces.)

2 Mechanics

1. **Newton's laws.** Make sure you draw on a diagram all the physical forces on the body, and then relate them to the resulting acceleration. Don't mix up F with ma if you want to make sure the signs come out correctly.

2. **Inertial frames.** "Fictitious" inertial forces in non-inertial frames.

3. **Rotation.** Centrifugal force (or centripetal acceleration) as an inertial force. Angular momentum and angular velocity: $L = I\omega$. Derivation of moments of inertia, I, for rods, discs and spheres.

4. **Gravity.** Circular orbits. Geostationary orbits. Escape velocity.

5. **Simple harmonic motion.** $m\ddot{x} + kx = 0$
Recognise and solve the differential equation for SHM in different contexts. Natural frequency ω. Derive general form for $x(t)$ and $\omega^2 = k/m$. You should be able to recognise the equations of motion for SHO in other situations outside mechanics, *e.g.* in circuits.

6. **Damped harmonic oscillator.** $m\ddot{x} - \gamma\dot{x} + kx = 0$
Underdamping, critical damping, and overdamping. Derive the behaviour of $x(t)$ in the three regimes (oscillatory *vs.* exponential decay.) Q factor of a resonance.

7. **Special Relativity.** Lorentz transformation for 4-vectors: (x, t) and (p, E). Lorentz contraction and time dilation (*e.g.*, muon decay lifetime). Transformation for velocity. Rest mass. Derivation of kinetic energy in the non-relativistic limit ($v \ll c$). Notion of simultaneity and causality in special relativity.

3 Waves

1. **Travelling waves and standing waves.** $\sin(kx - \omega t), \cos(kx - \omega t), e^{i(kx-\omega t)}$. Quantisation of k for standing waves.

2. **Principle of superposition.** Beats. Relation to Fourier analysis.

3. **Wave velocities.** Phase velocity ω/k and group velocity $d\omega/dk$. Use latter for flow of physical quantities, *e.g.* energy/information.

4. **Doppler effect.** For waves in a medium. For electromagnetic waves (photons).

5. **Diffraction: narrow slits.** Double slits and many slits (*i.e.* diffraction grating). You should be able to derive the condition for constructive interference: $n\lambda = d \sin\theta$, and to sketch the diffraction patterns for the two cases as a function of the angle of diffraction θ.

6. **Diffraction: single slit/finite diffraction grating.** Effect of finite slit width for double slit. Effect of finite number of slits for diffraction grating. Implication for theoretical resolution of optical instruments.

7. **Refraction.** Explain refraction as a consequence of change in velocity. Snell's law.

4 Quantum Mechanics

1. **DeBroglie relations.** $p = \hbar k$ and $E = \hbar\omega$. What do we mean by this?

2. **Heisenberg uncertainty principle: momentum and position.** $\Delta p\,\Delta x \geq \hbar/2$. Explain the meaning of Δp and Δx. Relationship to single-slit diffraction. Implication for the kinetic energy of confined particles.

3. **Heisenberg uncertainty principle: energy and time.** $\Delta E\,\Delta t \geq \hbar/2$. Implication for linewidths in atomic spectra.

4. **Observables & operators.** Observables are represented by Hermitian operators. Each operator has a set of eigenfunctions and eigenvalues. When this observable is measured, the outcome has to be one of these eigenvalues. The wavefunction for the system becomes the eigenfunction corresponding to this eigenvalue ("wavefunction collapse").

5. **Operators for momentum, kinetic energy and total energy.** $\hat{p} = -i\hbar\,\partial/\partial x$. What are the eigenfunctions of the momentum operator? Hamiltonian operator : $\hat{H} = \hat{p}^2/2m + V(\mathbf{r})$. Eigenstates and energy levels.

6. **Schrödinger equations.** Time-dependent and time-independent forms. Derive the time-independent form from the time-dependent form.

7. **1D scattering.** Transmission and reflection at a potential step or a potential barrier. Classically forbidden region. Tunnelling through a barrier: dependence on barrier width.

8. **1D potential wells.** Bound states in a infinitely deep potential well and in a finite potential well. Compare the eigenstates and energy levels in these two cases. Bound states in a double well: tunnelling and energy level splitting.

9. **Simple harmonic motion.** Quantum oscillator in a parabolic well: $E_n = \hbar\omega_0(n + 1/2)$. Sketch wavefunctions for the different energy levels.

5 Electromagnetism

1. **Maxwell's equations.** You are expected to remember these four equations.

2. **Gauss' law.** Electric field due to static charges. Applications: point charge, line charge, charged plane, charged sphere (inside and outside).

3. **Ampère's law.** Magnetic field due to steady currents. Applications: field around a current-carrying wire, inside a long solenoid.

4. **Faraday's law.** Electric field around a loop due to time-varying magnetic flux through the loop. Application

5. **Electrostatic potential.** Deriving electric field from the potential and *vice versa*: $\mathbf{E} = -\boldsymbol{\nabla} V$. Application: electostatic potential energy of a charged sphere.

Equipotentials and the method of images: charge near metallic sheet, metallic sphere.

6. **Force on charge.** $\mathbf{F} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$, *e.g.*, cyclotron motion in a uniform magnetic field.

7. **Dielectric.** Permittivity $\epsilon = \epsilon_r\epsilon_0$ and electric polarizability.

8. **EM wave in vacuo.** $c^2 = 1/\mu_0\epsilon_0$. Polarisation: linear/circular. In free space, $\mathbf{E} \perp \mathbf{B}$ and both are normal to direction of propagation.

9. **Poynting vector.** $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0$ gives energy density current (energy flowing through unit area per unit time). Radiation pressure: $p = 2S/c$ for a perfectly reflecting mirror.

10. **Metals: Ohm's Law.** Conductivity σ and resistivity ρ. Relation between resistivity and resistance R. *Ohm's law*: $\mathbf{J} = \sigma\mathbf{E}$ where $\mathbf{J}$ is the current density (charge flowing through unit area per unit time).

11. **EM wave in dielectric.** Refraction: Snell's law, total internal reflection. Brewster angle.

12. **EM wave in conductors.** Skin depth: absorption of EM wave in a conductor.

13. **EM wave in waveguides.** TE and TM modes.

6 Thermodynamics and Statistical Physics

1. **Basic concepts.** Heat and reservoirs. Entropy. Reversibility. State variables.

2. **Laws of thermodynamics.** 1st law: energy conservation $dU = dQ - dW$. (Define U, Q and W carefully to get signs right.) 2nd law: $dS \geq 0$.

3. **Fundamental equation.** $dU = TdS - pdV$.

4. **Equations of State.** Ideal gas: $pV = NkT$. Van der Waals gas.

5. **Simple processes.** Work done, heat exchanged, entropy change during adiabatic, isothermal, free expansions.

6. **Engines.** Carnot cycle. Definition of efficiency η. Efficiency of Carnot engine.

7. **Free energies.** Free energies as functions of state variables. Internal energy $U(S,V)$, Helmholtz $F(T,V) = U-TS$, Gibbs $G(T,p) = U-TS+pV$. Expressions for dF and dG. Maxwell relations.

8. **Classical statistical physics.** Statistical definition of entropy: microstates and macrostates. Boltzmann distribution for classical particles.

9. **Quantum statistics.** Identical particles: bosons and fermions. Pauli exclusion principle and the Fermi energy. Bose-Einstein and Fermi-Dirac distributions. Density of states. Application: black-body radiation, heat capacity.

7 Atomic Physics

1. **Spectral lines.** Use of spectral lines to understand atomic energy levels. Broadening of spectral lines: intrinsic, Doppler, collisions.
2. **Hydrogen atom.** Principal quantum number n and angular momentum l. Energy levels: s, p, d, f,. . ..
3. **Vector model.** LS coupling: total angular momentum: J.
4. **Selection rules.** Concept.
5. **Perturbation theory.** Application of first-order perturbation theory in atomic physics.
6. **Zeeman effect.**
7. **Molecules.** Bonding. Rotational modes and infra-red spectroscopy.
8. **Lasers.** Spontaneous and stimulated emission. Population inversion.

8 Electrons in Solids/Solid State Physics

1. **Free electron model.** Counting states in k-space. Density of states $g(E)$. Fermi level E_{F} & Fermi wavevector k_{F}. Metals: dependence of physical properties on $g(E_{\mathrm{F}})$, *e.g.* electronic heat capacity.
2. **Electron bands in solids.** Qualitative discussion of the formation of electrons bands from localised orbitals. Particles and holes. Effective mass.
3. **Bloch's theorem.** Electrons in a periodic potential. Reciprocal space: Brillouin zone.
4. **Insulators, semiconductors and metals.** Explanation in terms of the Fermi level relative to the position of electron bands. Importance of band gaps.
5. **Semiconductors.** Intrinic carrier density by thermal properties. Extrinsic carriers: n-type and p-type doping.
6. **p-n junction.** Physics at the interface: shifted electron bands, depletion layer. Application as diode: current-voltage characteristic, forward and reverse bias, breakdown.

7. **Optical properties of semiconductors.** p-n junction as a light-emitting diode or light detector. Relation between the band gap and the photon energy.

9 Particle and Nuclear Physics

1. **Fundamental particles.** Particles and antiparticles.
2. **Nuclear decay.** Alpha, beta, gamma decays.
3. **Nuclear reactions.** Fission and fusion: basic concepts of mass defect.
4. **Nuclear stability.** Semi-empirical mass formula: origin of the terms and their influence of the stability of the nucleus. Nuclear shell model.

10 Probability and Statistics

1. **Probability distributions.** Discrete distributions: probabilities P_i with normalisation $\sum_i P_i = 1$. Continuous distributions: probability densities $p(x)$ with $\int p(x)dx = 1$. The sum/integration is taken over all possible values of the random variable x.
2. **Expectation values.** For random variable x, the expectation value of any function $f(x)$ is $\langle f(x)\rangle = \sum_i f(x_i)P_i$ for discrete distributions, $\int f(x)p(x)dx$ for continuous distributions.
3. **Mean.** $\mu = \langle x\rangle = \sum_i x_i P_i$ for discrete distributions, $\int xp(x)dx$ for continuous distributions.
4. **Variance.** $\mathrm{var}\, x = \langle (x-\mu)^2\rangle = \langle x^2\rangle - \langle x\rangle^2$. Standard deviation: $\sigma = (\mathrm{var}\, x)^{1/2}$.
5. **Normal/Gaussian distribution.** $p(x) = (2\pi\sigma^2)^{-1/2}\exp[-(x-\mu)^2/2\sigma^2]$, *e.g.* for the result of an experimental measurement of a continuous variable with expected value μ and error σ.
6. **Binomial distribution.** Probability of M success out of N trials with probability p for success in an individual trial. Mean and variance.
7. **Poisson distribution.** $P(n) = e^{-\mu}\mu^n/n!$ for a rare event to occur n times if it occurs on average μ times. Variance = mean. *e.g.* number of nuclear decays in a given interval of time among a large collection of radioactive nuclei.
8. **Independent events/measurements.** The mean/variance of the sum is the sum of the means/variances. So, suppose $X = \sum_{j=1}^{N} x_j$ is the sum of N outcomes for the same random variable x, then $\langle X\rangle = N\langle x\rangle$ and $\mathrm{var}\, X = N\,\mathrm{var}\, x$.

9. **Central Limit Theorem.** For large N, the sum $X = \sum_{j=1}^{N} x_j$ has a Normal distribution with mean $N\mu$ and variance $N\sigma^2$ if x has mean μ and variance σ^2, *irrespective* of probability distribution for x.

10. **Estimating the mean from data.** For N independent measurements of x, best unbiased estimate of μ is $\bar{x} = N^{-1} \sum_{j=1}^{N} x_j$. Fractional error $\propto 1/N^{1/2}$.

11. **Estimating the variance from data.** Unbiased estimate of variance is $[N/(N-1)][N^{-1} \sum_{j=1}^{N} x_j^2 - \bar{x}^2]$.