Charged Particle in Magnetic Saddle Point

BSc Project 2002

This project studies the motion of a charged particle in two dimensions when a spatially varying magnetic field is applied perpendicular to the plane. In particular, we want to study how the particle trajectories behave near saddle points in the magnetic field. We can ask whether this dynamics is chaotic and analyse the statistics of the trajectories. We can also study the quantum analogue of this problem and examine how quantum chaos is exhibited in this system.

I. GETTING STARTED

We study charged particles moving in two dimensions. From the perspective of solid state physics, this can be realised for electrons in semiconductor quantum wells. For the moment, we will ignore the quantum nature of the electron, and study classical dynamics in two dimensions.

Now apply a perpendicular magnetic field, $B(\mathbf{r})$. For a uniform field, a particle of mass m and charge e goes round in circles at a frequency $\omega = eB/m$. If the initial momentum is p, then the radius of the orbit is p/eB.

- Note that the Lorentz force is always perpendicular to the velocity, and so no work is done on the particle — its kinetic energy $p^2/2m$ is a constant of motion.
- What happens to the orbits as $B \to 0$?

Consider now a magnetic field with a constant gradient:

$$B(\mathbf{r}) = b'y \tag{1}$$

You should satisfy yourself that:

- away from the $B = 0$ line, a particle drifts in the x-direction with a drift velocity proportional to b'
- a particle that crosses the $B = 0$ drifts too. It is possible to drift in both $\pm x$ directions.

Task: Write a program to compute the trajectories for this case numerically from the equation of motion. This will be the "experimental tool" for most of this project.

- The code should allow for flexibility, *e.g.* arbitrary magnetic fields, initial conditions, size of the time step, . . .

- Look out for numerical instabilities. Since we are going to study what looks like chaos, we should separate randomness from the physics and noise from the computer!
- Check that the trajectories you get are what you expect for the example of a constant field gradient.

If you want some analytical work, come and ask me about how to calculate the period for the trajectories in this case.