Comment on "Self-Organized Criticality in a Continuous, Nonconservative Cellular Automaton Modeling Earthquakes"

In a recent Letter Olami, Feder, and Christensen [1] investigated the scaling behavior of a cellular automaton model of an earthquake fault. Their two-dimensional model consists of blocks with initial force F_{ij} distributed at random. If F_{ij} exceeds a spatially constant threshold $F_{\rm th}$ then the block fails.

Failure consists of giving αF_{ij} ($\alpha = \text{const} \leq 0.25$) units of force to each neighboring block and setting the force on the failed block equal to zero. If $F_{ij} < F_{\rm th}$ for all blocks of the system the algorithm proceeds by locating the block with the largest force $F_{\rm max}$ and incrementing the force on each block of the system by an amount $F_{\rm th} - F_{\rm max}$. The process of checking for $F_{ij} \geq F_{\rm th}$ then repeats.

Olami, Feder, and Christensen count the number of failure events after each global force increment. They count each failure of a block as an event. If a block fails n times, it is counted as n failure events. In general a block may fail several times after one global force increment. Arguing that the energy released in an "earthquake" event is proportional to the number of failure events, Olami, Feder, and Christensen display a log-log plot of the probability of an event of energy E versus the energy E.

The data have two distinct features. First, the probability versus energy plots are straight lines with apparent exponential cutoff but a slope that depends on α. Second, the exponential cutoff is claimed to scale with the linear size of the system L as $L^{2.2}$. We contend that the scaling of the cutoff cannot be $L^{2.2}$, that this scaling leads to a contradiction. It is, of course, obvious that if sites can only fail once, this cutoff scaling is impossible, but we contend that even if sites have multiple failures, the cutoff scaling is contradictory.

To see this we note that after the first implementation of the algorithm the force in the system (for two dimensions) is bounded by $F_{\rm th}L^2$. If the exponential cutoff scales as $L^{2.2}$ then there is a nonzero probability that after some global force increment an "earthquake" with $L^{2.2}$ failure events occurred. Since the amount of force removed from the system for each failure is bounded from below by $(1-4\alpha)F_{\rm th}$, the force F_R removed from the system by $L^{2.2}$ failures is $F_R \geq (1-4\alpha)F_{\rm th}L^{2.2}$. But for sufficiently large L, $(1-4\alpha)F_{\rm th}L^{2.2} > F_{\rm th}L^2$ and this is obviously a contradiction.

It is instructive to estimate the value of L for which this contradiction becomes obvious numerically. For $\alpha = 0.2$, the value used in Olami, Feder, and Christensen, the critical value can be estimated by $L^2 = (1-0.8)L^{2.2}$ or $L = 3125$. Since the largest L checked by Olami, Feder, and Christensen is $L = 50$, their data are far from this limit. However, one must conclude that if the data scale as stated in Ref. [1], in spite of the contradiction, then the system has not reached the true scaling limit and conclusions about critical behavior are suspect.

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[1] Z. Olami, H. J. S. Feder, and K. Christensen, Phys. Rev. Lett. **68**, 1244 (1992).

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