Micro rain radar measurements reveal evidence of self-organised criticality in precipitation processes

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I. INTRODUCTION

Slowly driven dissipative systems displaying scale-free energy releases over wide ranges have been termed self-organised critical (SOC). For an introduction see: Jensen, 1998. Simple cellular automaton models are found to obey the simple scaling ansatz $P(s, L) \propto s^{-\tau} G(s/s_\xi)$, where $P(s, L)$ denotes the probability of an energy release of size s, G is a scaling function, whose argument depends upon the system size L through the cut-off size $s_\xi \propto L^D$. G is constant for small arguments ($s/s\xi \ll 1$), and then decays rapidly. The most studied models of this type are so-called "sandpiles" for which the critical exponents are known numerically, and analytically for the simplest versions. The models are defined on discrete lattices to which energy units are added at random positions. Local interaction rules implement a threshold of energy density. When the threshold is surpassed somewhere in the system, energy is redistributed away from the overloaded site. This redistribution can cause avalanches to propagate through the system. If dissipation of energy takes place only at the system boundaries, then avalanches of all sizes are observed up to a cut-off set by the system size L through the argument of the scaling function. The above scaling ansatz emerges in the context of closed systems tuned to the critical point of a phase transition. The fact that SOC models do not contain any tunable parameters and yet obey the critical scaling ansatz explains the term "self-organised criticality". To ascertain criticality, the cut-off size s_ξ (apart from the size of individual lattice sites) must be shown to scale with the system size L. While this can be done in cellular automaton models, in real-world systems we often have no control over the system size. But when the likelihood of extreme events defies intuition, when spatio-temporal correlation exists on surprisingly large scales, when scale-freedom is observed over wide ranges, lessons may be learned from the knowledge we have of the deceptively simple "sandpiles".

II. THE ATMOSPHERE AS A SANDPILE

The following features are common to all SOC models: They are open systems, slowly driven by an energy input; energy can be stored intermediately, local energy density thresholds exist, and dissipation from high-energy regions is fast. The atmosphere can be thought of as such a system (Peters, O. *et al.* 2002). It is slowly driven by solar radiation, which pumps water vapour into the system, representing the evaporation energy. This energy is stored in the atmosphere until local energy density thresholds, the local saturation levels, are surpassed. Dissipation is then fast: Energy leaves the system in the form of rain showers. In 6 months of data we found the longest rain event to be of the order of one thousand minutes, while the longest dry period was more than one order of magnitude longer.

III. RAIN EVENT SIZES

To find out whether the atmosphere might be an example of a self-organised critical system, we investigate a time series of rainfall from the Baltic Sea coast with one minute temporal resolution. Data were collected with a micro rain radar (MRR2) (Peters, G. *et al.* 2002), developed by METEK GmbH and operated by the Max-Planck-Institute for meteorology in Hamburg.

The most striking feature of an SOC system is the scale-free, i.e. power-law, distribution of avalanche sizes discussed in Sec. I. To employ the SOC perspective we therefore need a definition of an "avalanche", i.e. a rain event. Previous work focused on rainfall during fixed time intervals and on the statistical properties of such fluctuating rain intensities. In contrast, we would like to let the system choose the observational time scale and use the event as the fundamental building block of the process. Fundamentally, rain consists of very short events, the impact of rain drops. These events last about a millisecond and the rain rates are of the order of 10^7 mm/h (the fall velocity of a rain drop). At the other end of the spectrum as we coarse-grain our observations spatially we reach global scales at which it rains constantly, with a rate nearing that of global evaporation. Apart from spatial and temporal resolution of our observations, we also need to take into account another "coarseness": the rain gauge sensitivity. Structures observed in precipitation data thus depend crucially on three scales: time, space, and sensitivity. To start out we define the rain event as a sequence of non-zero rain rates $q(t)$ in our time series, and its size $M = \sum_t q(t)\Delta t$, with $\Delta t = 1$ min, is the accumulated water column during the event. The intervals of zero rain rate between events are called dry periods.

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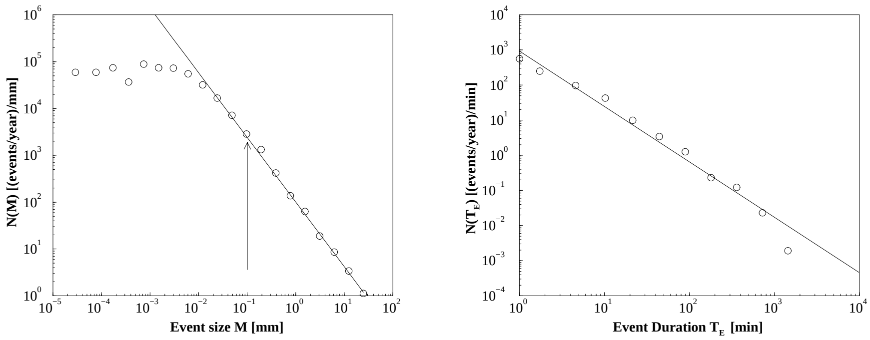

FIG. 1: Left: The number density N(M) of rain events versus the event size M (open circles) on a double logarithmic scale. Events are collected in bins of exponentially increasing widths. The horizontal position of a data point corresponds to the geometric mean of the beginning and the end of a bin. The vertical position is the number of events in that bin divided by the width of the bin. To facilitate comparison with future work, we rescaled the number of events to annual values. The experimental data are consistent with a power law $N(M) \propto M^{-\tau_M}, \tau_M \approx 1.4$ (solid line) over at least three orders of magnitude, $M \epsilon [M_{min}, M_{max}]$ with $M_{min} \approx 5 * 10^{-3}$ mm and $M_{max} \approx 35$ mm. The arrow indicates the typical sensitivity threshold of a conventional high precision tipping bucket rain gauge. Not only can we see that the radar technique is roughly 10,000 times more precise but also that a considerable fraction of rain events must be missed with conventional methods.
Right: If rain fell at a constant rate, the duration of an event would be proportional to the size. For the frequency distribution we would expect a power law with the same exponent. This is not observed. Instead, longer rain events in our time series were more intense.

Figure 1 shows the number density of rain events per year N(M) versus event size M on a double logarithmic plot. In a scaling regime $M_{min} < M < M_{max}$ extending over at least three orders of magnitude, the distribution follows the simple power law $N(M) \propto M^{-\tau_M}, \ \ \tau_M \approx 1.4, \ \ M \epsilon [M_{min}, M_{max}]$. This is the typical "critical" dynamical behaviour found in SOC systems. For events smaller than $M_{min} \approx 5 * 10^{-3}$ mm the power law breaks down. This is indicative of a different physical process being responsible for events in this realm. Given that this interpretation of is correct, and every rain event with $M > 5 * 10^{-3}$ mm is actual rain, measurements with today's standard precision simply don't see the small rain events, which are by far the most frequent. In our case, the events with $M_{min} < M < 0.1$ mm constitute 68% of the events within the scaling region. While these smallest 68% of all proper rain events do not account for a large amount of water, they are invaluable in addressing questions regarding the fundamental structure of rain and the actual physical processes involved.

With the radar measurement, all rain seems to be captured and we can choose a suitable limit (M_{min}) below which events are ascribed to a different physical process. To ascertain that we are capturing the entire physically relevant range of the observables of the process of rain, it is necessary to use observational techniques enabling us to see beyond the physical limits of rain. Results from investigations that do not fulfill this requirement cannot be conclusive and must be treated with careful scepticism. The present study suggests a reasonable maximum sensitivity threshold of around $5 * 10^{-3}$ mm, which is one twentieth of the commonly used threshold.

From extrapolating the power law we expect to see 38 events larger than the largest actually observed. We therefore believe that the upper cut-off apparent in Fig. 1 is not due to the limited time of observation but rather reflects a physical limit to the process of rain at the given location. We define M_{max} as the largest event in the data set, a downpour of $M_{max} \equiv 35$ mm of rain. Confirmation of this cut-off by longer time series, and the incorporation of spatial information could be uselful to assess maximum probable precipitation and floods.

IV. DRY PERIODS

A power law in the frequency distribution $N(T_D)$ of waiting times between individual events has been observed previously and can be related to the fractal dimension of the binary rain/ no rain signal at a given location. While in Fig. 2 (left) we don't see any evidence of a breakdown of this power-law, the box counting fractal dimension in Fig. 2 (right) shows a change of behaviour at a time scale of order 10 min. Note that the box-counting fractal dimension contains information about the order of dry periods, while the frequency distribution $N(T_D)$ does not. We note that none of our results were significantly affected by temporal coarse graining with to 10 min resolution. At much lower temporal resolution (60 min), events merge, and the critical behaviour begins to wash out.

The explanation for the upper cut-off of the fractal regime may be that the typical duration of a frontal system moving in from the Atlantic is of the order of 3 days. Measured rain parameters will not belong to the same frontal system if the measurements are temporally separated by significantly more than three days.

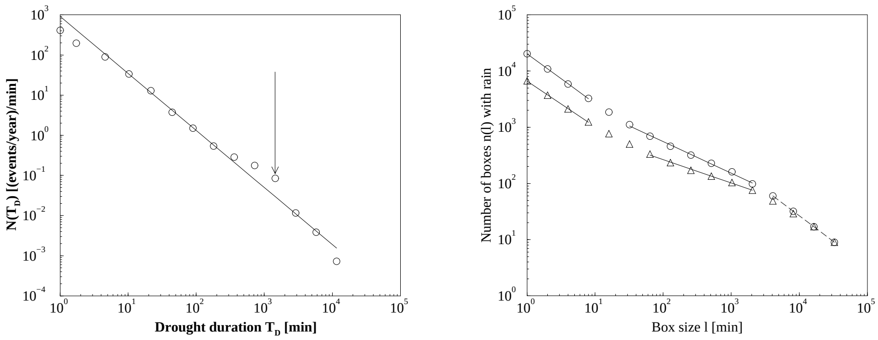

FIG. 2: Left: The open circles show the number density $N(T_D)$ of dry periods per year versus their duration T_D. The solid line represents a power law approximation, $N(T_D) \propto T_D^{-\tau_D}, \tau_D \approx 1.4$, to the observed distribution. The arrow indicates one day, around which a deviation from pure power law behaviour can be observed. This is due to the daily meteorological cycle. Right: The number of time intervals (boxes) needed to cover the rain versus the box size l. The fractal dimension d_f is minus the slope of this graph in a double logarithmic plot. Using the radar sensitivity as our threshold value to distinguish between rain and drought (circles), we obtain $d_f \approx 0.55$ in a scaling regime spanning 2 orders of magnitude. Introducing an arbitrarily chosen higher threshold of 2.1 mm/h, corresponding to 420 times the sensitivity threshold of the radar, we observe a lowering of the fractal dimension to $d'_f \approx 0.42$ (triangles). For values of l to the left of the central scaling regime, it assumes the value $d_f \approx 0.88$ without an artificial threshold and $d'_f = 0.81$ with 2.1 mm/h threshold. To the right of the central scaling regime the fractal dimension approaches the trivial value $d_f = 1$.

Measurements with a sensitivity threshold greater than 0.003 mm will yield results reflecting the instrument's sensitivity rather than properties of the dynamical system producing the rainfall. Similar multifractal behaviour has been observed in SOC models (Olami *et al.* 1992).

The lower breakdown indicates that 10 min to 30 min is a special time scale, and it must be related to a physical process. Starting with typical small cloud droplets with radius $r \approx 10^{-3}$ mm, the process of stochastic collection during which small droplets merge to form rain drops of appreciable fall velocity is believed to take $10 - 30$ min under typical warm cloud conditions (Houze 1993). Might this be the origing of the 10 min time scale?

V. CONCLUSION

Viewing rain events as SOC-type energy relaxation, scale-free power-law behaviour is found to govern the statistics of rain over a wide range of time- and event size scales. Deviations from the observed power laws indicate the limits and peculiarities of the underlying dynamical system, and physical insight is gained. Our findings suggest that rain is an excellent example of a self-organised critical process. For our purposes, the remote sensing technique employed by the MRR2 proved to be extremely powerful. It is our impression that a precondition for gaining a deeper understanding of the atmosphere as a complex dynamical system is the gathering of comprehensive data sets with such high precision instruments. To observe the reported aspects of atmospheric dynamics, temporal resolution of at most 10 min and a sensitivity of no more than 0.003 mm/h was necessary. Comparison with data from tropical sites with more convective rain as well as the incorporation of spatial structure would be useful in order to answer questions regarding the universality of the observed features. Analysis of data from a network of high-precision radars with emphasis on the spatio-temporal nature of the processes would be particularly instructive.

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