AVALANCHES IN PILES OF RICE

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Abstract. A quasi one-dimensional rice pile has been used to study the dynamics of driven, inhomogeneous systems. Grains of rice were slowly fed into the gap between two vertical plates, a pile built up and reached a quasi-stationary state. The collective transport properties in terms of avalanches was addressed. For elongated grains the probability density for avalanches (energy dissipation events) *within* the pile is a power law whereas more spherical grains lead to a stretched-exponential form. Thus the microscopic details determine whether the response in this self-organizing system is critical or not. Furthermore, we have studied experimentally the individual transport properties in the system displaying self-organized criticality. Tracer particles were added to the pile and their transit times measured. The distribution of transit times is a constant with a crossover to a decaying power law. The average transport velocity decreases with system size. This is due to an increase in the active zone depth with system size. This picture is supported by considering transport in a $1D$ cellular automaton modeling the experiment.

1. Introduction

The avalanches that occur when grains are dropped onto a pile illustrate the spontaneous generation of complexity in simple dynamical systems [1]. When grains are dropped onto a finite base, a pile builds up. However, it cannot become infinitely high, and, eventually, the system settles in a stationary state where the outflux over the edge of the base on average equals the influx. Intermittent flow of grains down the slope of the pile (small and large avalanches) maintain the system in this state. Bak, Tang, and Wiesenfeld constructed a $2D$ cellular automaton of a slowly driven dynamical system. They showed, that the 'pile' spontaneously evolves, or

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self-organizes, into a state with avalanches of all sizes distributed according to a power law, that is, there is no internal system-specific scale. Because of the lack of any characteristic avalanche size, the system is referred to as critical [1]. It has been a longstanding question whether real granular systems display self-organized criticality (SOC) when slowly driven. Recently, however, an experiment on a quasi one-dimensional pile of rice has shown that the occurrence of SOC depends on details in the grain-level dissipation mechanisms [2]. Only with sufficiently elongated grains, avalanches with a power-law distribution occurred.

2. The experiment

The experimental system consisted of a rice pile confined between two 5 mm thick glass plates supported by 15 mm thick 100 cm × 120 cm polymethylmethacrylate plates. Aluminum rods were inserted between the glass plates to form a vertical wall at one side and a variable base with length $\mathcal{L}$ of the quasi one-dimensional pile. The other side was open, allowing grains to fall off the pile. Grains of rice were slowly fed into the gap between the plates close to the vertical wall using a single seed machine. We used a plate separation to grain length ratio of approximately 0.8 and system sizes $\mathcal{L} = 15, 30, 60$, and 85.7 cm. The injection rate was 2–3 grains every 7.7 s or, on average, 20 grains/min.

2.1. THE COLLECTIVE BEHAVIOUR: AVALANCHES

The dynamics of the rice piles was recorded with a photometric CCD camera with 2000×2000 (pixels)^2 spatial resolution. Frames were taken at 15 s intervals and the profiles were identified. Each experiment lasted about 42 h and consisted of 10,000 profiles. Let $E^a(\text{pot})$ and $E^b(\text{pot})$ denote the potential energies associated with two consecutive profiles a and b. If $E^{add}(\text{pot})$ is the potential energy and $E^{add}(\text{kin})$ the kinetic energy of the added grains, then the energy E dissipated in the rice pile by an avalanche is given by

$$\begin{aligned} E &= E^a(\text{pot}) + E^{add}(\text{pot}) + E^{add}(\text{kin}) - E^{out}(\text{kin}) - E^b(\text{pot}) \quad (1) \\ &\approx E^a(\text{pot}) + E^{add}(\text{pot}) - E^b(\text{pot}) \end{aligned}$$

since the kinetic energies of the incoming grains $E^{add}(\text{kin})$ and grains dropping out (if any) $E^{out}(\text{kin})$ can be neglected.

Figure 1 is a finite-size scaling plot of the probability density $P_E(E, L)$ for elongated rice grains – aspect ratio ≈ 3.8 – and system sizes $L = 16$, 33, 66, and 105 (in terms of the grain length δ), marked with lines of increasing dash length. The energy dissipation is expressed in terms of $\varepsilon = mg\delta = 1.54\,\mu$J where m is the average grain mass, g the acceleration

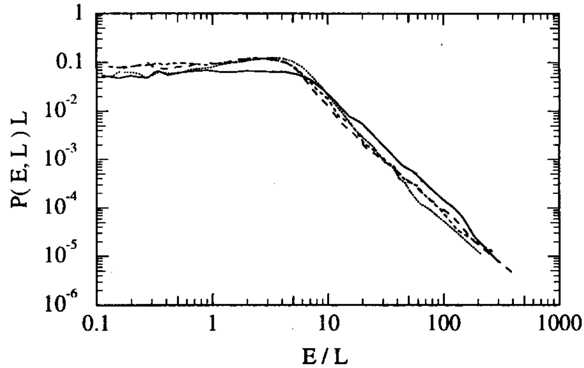

Figure 1. A finite-size scaling plot of the avalanche size distribution for the elongated grins using $P_E(E, L) = L^{-\beta_E} f(E/L^{\nu_E})$ with $\beta_E = \nu_E = 1$. The scaling function has the form $f(x) = \text{const}$ for $x < 1$ and $f(x) \propto x^{-\alpha_E}$ for $x > 1$, with $\alpha_E \approx 2.04$.

of gravity and δ the grain length δ. The probability densities are constant for 'small' values of E and have a power-law form $P_E(E, L) \propto E^{-\alpha_E}$, extending over approximately one and a half decade, for 'large' E, thus there is no characteristic avalanche size. There is no obvious cutoff in the scaling function, the power-law behaviour extends up to the largest avalanche sizes measured. In the scaling argument, a cutoff is unnecessary when the power-law exponent $\alpha_E > 2$. The crossover to a constant probability density is caused by an uncertainty in the measured energy dissipation, $\Delta E \approx \Delta E^{add}(\text{pot}) \propto \Delta N L$, where ΔN is the uncertainty in the no. grains added between two profiles. We expect to observe a larger scaling region in a more careful experiment where only one grain is added at a time, leaving $\Delta E \approx 0$. Thus the avalanche dynamics in the rice pile with the elongated grains is consistent with a self-organized critical process.

For more spherical grains, – aspect ratio ≈ 2.0 – we find that the probability densities are consistent with a stretched-exponential scaling function, $f(x) \propto \exp(-(x/x^*)^\gamma)$ with $\gamma \approx 0.43$ and $x^* \approx 0.45$. A characteristic avalanche size $E^* = x^* L$ for the dynamics of the more spherical grains appears, which is inconsistent with the idea of SOC. The more spherical grains tended to roll down the slope which resulted in a very small grain to grain friction rendering inertia effect important. A large aspect ratio leads to a 'rough' profile and a sliding grain motion with a higher effective friction which seems crucial to get 'critical dynamics'.

2.2. THE INDIVIDUAL BEHAVIOUR: TRACER PARTICLES

The elongated grains could pack in a variety of ways, and each avalanche replaced, locally or globally, one surface configuration with another. Thus a

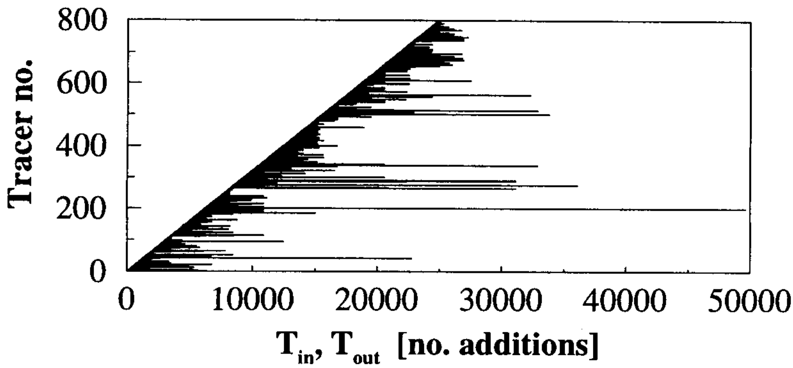

Figure 2. A record of the tracer experiment in a pile of size $L = 113$ where a total number of 800 tracer particles were added, one every 4th minute. The tilted line connects the injection times for all the tracers. The transit time for each tracer particle is represented by the length of a horizontal line whose projection onto the x-axis of the left (right) endpoint equals the time the particle entered T_{in} (left T_{out}) the system. The transit time is measured in units of the number of injections of uncolored grains (no. additions), 1 addition every 7.7 s. Note the large variation in the transit times $T = T_{out} - T_{in}$ and that, repeatedly, many tracers left the system at the same time.

dynamically varying medium disorder (coupled to the relaxation processes) was generated. This is conceptually different from transport in media with a quenched disorder. Furthermore, in SOC systems, a small perturbation may lead to arbitrarily large avalanches, and it is not clear at all, how this affects the transport properties. Thus it is quite surprising, that there are no experiments and only a few theoretical and numerical studies on transport in systems displaying SOC. In collaboration with Álvaro Corral, Vidar Frette, Jens Feder and Torstein Jøssang I have studied experimentally transport properties in the slowly driven rice pile with elongated grains which display SOC [3]. Tracer particles were added to a pile and their transit times measured, see Figure 2.

Figure 3 shows the resulting statistics of the data in a finite-size scaling plot with $P_T(T,L) = L^{-\beta_T} g(T/L^{\nu_T})$. The distribution of transit times is a constant with a crossover to a decaying power law with exponent $\alpha_T \approx 2.4$.

Since $\alpha_T > 2$, the average transit time $\langle T \rangle \propto L^{\nu_T}$ where L is the system size and $\nu_T = 1.5 \pm 0.2$. Thus the average velocity of tracer particles $\langle V \rangle \propto L/\langle T \rangle$ decreases with system size. This is due to an increase in the active zone depth with system size. The number of particles crossing an active zone λ_L in a fixed time interval δt, $\delta t \langle V \rangle \lambda_L$, is proportionally with the constant rate of adding particles. Thus $\langle V \rangle \propto 1/\lambda_L$.

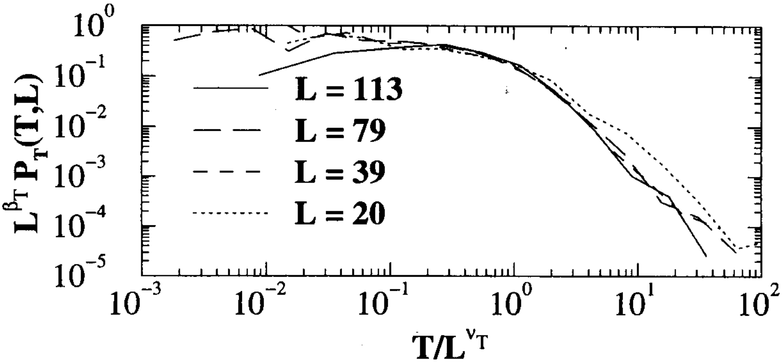

Figure 3. A finite-size scaling plot of the experimental results for the normalized distribution of transit times in piles with sizes $L = 20, 39, 79$, and 113. The data have been averaged over exponentially increasing bins with base 2 in order to reduce the fluctuations in the statistics due to the relatively small number of tracer particles. Disregarding the smallest system, a reasonable data collapse of the three largest systems is obtained with $\nu_T = 1.5 \pm 0.2$ and $\beta_T = 1.4 \pm 0.2$, The scaling function g is essentially constant for small arguments and have a decaying power-law tail with a slope of $\alpha_T = 2.4 \pm 0.2$. These large transit times correspond to tracer particles which, during the transport through the systems, become deeply embedded in the pile.

3. The model

Inspired by the experiments, we considered a model of size L in which an integer variable h_x gives the height of the pile at site x. The local slope z_x at site x is given by $z_x = h_x - h_{x+1}$ where we impose $h_{L+1} = 0$. The addition of a grain at the wall increases the slope by one at $x = 1$, that is, $z_1 \to z_1 + 1$. We proceed by dropping grains at the wall until the slope z_1 exceeds a critical value, $z_1 > z_1^c$, then the site topples by transferring one grain to its neighboring site on the right. If $z_x > z_x^c$, this site topples in turn according to $z_x \to z_x - 2, z_{x\pm1} \to z_{x\pm1} + 1$ (unless at the rightmost site where the grains fall off the pile) generating an avalanche. During the avalanche, no grains are added to the pile. Thus the two time scales involved in the dynamic evolution of the pile are separated. The injection rate of grains is low compared to the duration of the relaxation processes. The avalanche stops when the system reaches a stable state with $z_x \leq z_x^c \,\forall x$ and grains are added at the wall until a new avalanche is initiated and so on. The critical slopes z_x^c are dynamical variables chosen randomly to be 1 or 2 every time site x has toppled. This is a simple way to model the changes in the local slopes observed in the rice pile experiment. Thus the model differs from the trivial $1D$ BTW model where $z_x^c = 1$ is a constant [1]. Starting with, say, $z_x = 0$ and $z_x^c = 1 \,\forall x$, the system reaches a stationary state where the avalanche sizes are power-law distributed with an exponent of -1.55 ± 0.10

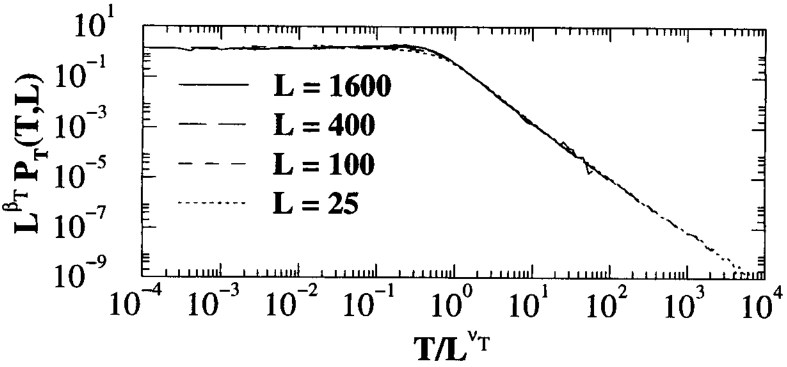

Figure 4. A finite-size scaling plot with $\nu_T = 1.30 \pm 0.10$ and $\beta_T = 1.35 \pm 0.10$ of the normalized distribution of transit times in the numerical model with system sizes $L = 25, 100, 400$, and 1600. The statistics shown correspond to 10^7 tracer particles (10^6 for $L = 1600$), and the data have been averaged over exponentially increasing bins with base 1.1. The functions are constant for small transit times and decay as power laws with a slope of $\alpha_T = 2.22 \pm 0.10$.

and a cutoff in the power-law distribution that scales with system size as $L^{2.25\pm0.10}$.

When the system has reached the statistically stationary state we measured the transit times of all the added particles in the model as a function of system size, see Figure 4. For further details please see [3].

In conclusion, this new direction of research sheds light upon the dynamics of SOC systems in general and granular systems in particular. We find that the transport properties of a SOC granular medium are characterized by an average velocity that approaches zero when the system size increases. These experimental findings agree well with the behavior seen in a simple $1D$ computer model of the self-organized critical pile.

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