Comment on "Universal Fluctuations in Correlated Systems"

In [1] the "universal" probability density function (PDF) of various global measures x from correlated equilibrium and nonequilibrium physics-inspired models was shown (Fig. 2 of [1]) to be in reasonable agreement with $\Pi(y) = K(e^{y-e^y})^a$, where $y = b(x - s)$. This Bramwell-Holdsworth-Pinton (BHP) PDF with shape parameter $a = \pi/2$ was contrasted with the Fisher-Tippett-Gumbel (FTG) PDF where $a = 1$, one of three possible asymptotes for the maxima of uncorrelated random variables (r.v.'s) [2]. In [1] the BHP PDF was also seen to describe the distribution of maxima $\xi^L_{\max}$ from L trials of length N vectors ξ of correlated r.v.'s, generated by $\xi = \mathbf{M}\chi$. The elements of χ were independently and identically distributed (i.i.d.) with exponential PDF, and $\mathbf{M}$ was "an $N \times N$ matrix with random but fixed elements." The applicability of the BHP PDF to all the considered processes in [1] was inferred to be due to "finite size, *strong correlations* (our italics) and self-similarity." In this Comment we show that this inference is not supported by the extremal behavior of $\xi^L_{\max}$ because (1) while a suitable choice of N ensures that $\xi^L_{\max}$ falls on the BHP curve, other equally arbitrary values of N give maxima which do not and (2) ξ is in fact not strongly correlated, and the PDF of $\xi^L_{\max}$ change little for decorrelated ξ. We thus suggest that a differs from its FTG asymptote of 1 here largely due to slow convergence of maxima of the nearly Gaussian ξ.

In [1], "random" implies "i.i.d. Gaussian, mean 0, variance 1"; i.e., $P(M) = \frac{1}{\sqrt{2\pi}} e^{-M^2/2}$ and "random but fixed" implies $\mathbf{M}$ has random elements held constant between trials. ξ is correlated, but the fact that $\mathbf{M}$ and χ are statistically independent means that for the simplest case of $N = 2$, ξ_1 $(= M_{11}\chi_1 + M_{12}\chi_2)$ has PDF $P(\xi_1) = \int dM_{11} dM_{12} d\chi_1 d\chi_2 \delta(\xi_1 - M_{11}\chi_1 - M_{12}\chi_2) P(M_{11}) \times P(M_{12}) P(\chi_1) P(\chi_2)$. Taking exponential $P(\chi) = \lambda e^{-\lambda\chi}$ as in [1] we generalize to $N \times N$ matrix $\mathbf{M}$ so $P(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} I(k)^N e^{ik\xi} dk$. Here $I(k)^N = \lambda^N [\int_{-\infty}^{\infty} \frac{P(M)}{\lambda + ikM} dM]^N$ is the generating function of $P(\xi)$. We then have $I(k) = \sqrt{\frac{\pi}{2}} \frac{\lambda}{k} e^{\lambda^2/2k^2}[1 - \mathrm{erf}(\lambda/\sqrt{2}k)]$. The maxima of ξ are dominated by the tail behavior of $P(\xi)$, which from a small k expansion of I is seen to be Gaussian $\sim(\lambda/\sqrt{N}) \times \exp -\xi^2\lambda^2/4N$. The maxima of ξ will thus be in the basin of attraction of the FTG distribution. The asymptotic shape parameter a corresponding to $L, N \to \infty$ cannot be changed by short range dependence [2], but one expects corrections to a for finite N, L because of the logarithmically slow convergence of the maxima of Gaussians, and, sometimes, a change in effective degrees of freedom N due to correlation [2].

Figure 1 shows the PDF of $\xi^L_{\max}$ for $L = 10^5$ trials in two cases: $N = 40$ and 250, and for comparison the PDF of $\chi^L_{\max} = \max(\chi^L)$, when $N = 40$, to illustrate the expected rapid convergence of $\chi^L_{\max}$ to the FTG curve. Convergence of $\xi^L_{\max}$, however, slows with increasing N. By $N = 40$, convergence of $\xi^L_{\max}$ is noticeably slower than $\chi^L_{\max}$ and the PDF follows the BHP curve, while for $N = 250$ even slower convergence in $\chi^L_{\max}$ gives obvious deviation from the BHP form resembling that seen in Fig. 2 of [1]. For $N = 40$ the autocorrelation function of χ (not shown) drops fast by the first lag. Weak (less than ± 0.2) long-ranged oscillations *are* seen at all lags. Despite this, decorrelating ξ by shuffling the $L \times N$ matrix before taking the maximum of ξ changes the PDF of its maxima in a relatively minor way (see Fig. 1), confirming that correlation cannot explain most of the change from the FTG PDF here. Our result implies that, even though subsequent results may show that the BHP curve *can* result from strong correlation, it *need not*.

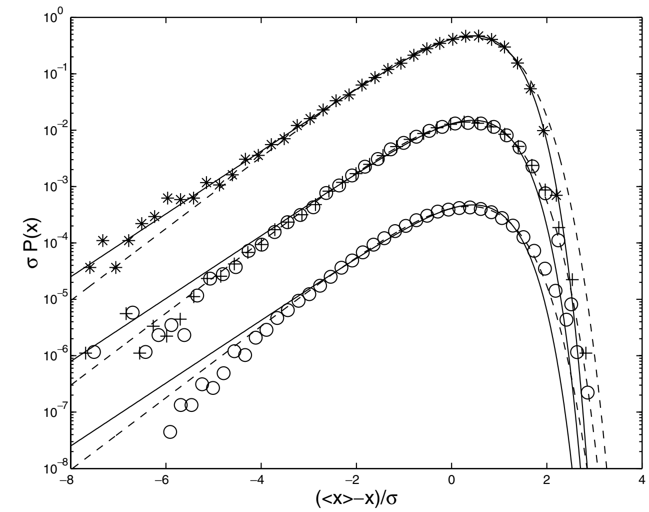

FIG. 1. $L = 10^5$. Middle plot shows the PDF of $\xi^L_{\max}$ (circles) compared with the PDF of maxima of decorrelated process (crosses), $N = 40$. Top plot (stars) is the PDF of maxima $\chi^L_{\max}$ of exponentially distributed r.v's, $N = 40$. Lowest plot (circles) shows $\xi^L_{\max}$ in the $N = 250$ case. Also shown on each plot are the BHP curve (solid line) and the $a = 1$ FTG plot (dashed line). Displacements of $10^{-1.5}$ and 10^{-3} for the middle and lower plots, respectively, were applied for clarity.

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