Construction of a DLA cluster model

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Received 13 December 1995

Abstract. On the occasion of the 50th birthday of a distinguished colleague, a three-dimensional wood model of a computer-generated DLA cluster was built. In this paper the design of the model and the construction process are described. The experiment may be carried out in the framework of a classroom experiment to demonstrate some of the fundamental concepts used in current research on growth phenomena. It is suitable as a first introduction to fractal geometry.

Zusammenfassung. Anläßlich des 50sten Geburtstag eines bedeutenden Kollegen wurde ein dreidimensionales Holzmodell eines computererzeugten DLA-Clusters gebaut. Hier beschreiben wir die Entwickelung des Modells und den Konstruktionsprozess. Das Experiment kann im Rahmen eines Klassenzimmerexperiments durchgeführt werden um einige der fundamentalen Konzepte innerhalb der aktuellen Forschung auf dem Gebiet der Wachstumsphänomene zu demonstrieren. Es eignet sich als eine erste Einführung in die fraktale Geometrie.

1. Introduction

The term ‘Diffusion Limited Aggregation’ (DLA) [1] refers to a simple growth algorithm in which individual particles are added to a growing cluster through a diffusion-like process. Starting from any suitable cluster seed, fixed in d-dimensional space, a new particle is launched at a random position, far away from the cluster, and undergoes Brownian motion. When the randomly walking particle contacts the cluster, it is stopped and incorporated into the cluster at its contacting position. This process of launching a random walker and adding it to the cluster on its first contact is repeated until the cluster has reached a desired number of particles (mass), s. The DLA model generates strongly ramified clusters with a complex geometry. Figure 1 displays a DLA cluster embedded in two-dimensional space at different stages of growth. The cluster consists of identical discs of diameter l. The cluster seed was a single disc.

Due to the non-deterministic nature of Brownian motion, each DLA cluster represents the individual outcome of a statistical process. Although the process itself is simple, its boundary conditions are not. The surface of the cluster constitutes the absorbing boundary for the diffusing particles and changes while the cluster grows.

More specifically, DLA is a growth process in which the growth rate of the cluster boundary is determined by the gradient of a scalar field Φ that fulfils the Laplace equation. The boundary conditions are that $\Phi = 0$ on the cluster boundary, and $\Phi = 1$ at infinity. The random walkers provide unbiased samples of the Laplace field Φ, and the probability of contacting the cluster boundary at a given position x is proportional to $\Phi(x)$. In summary, DLA is a model for Laplacian growth in the presence of statistical noise. Consequently, the DLA algorithm is a relevant model for many Laplacian growth phenomena in nature, including fluid–fluid displacement in Hele–Shaw cells [2] and in porous media [3, 4], bacterial growth [5], electrodeposition [6, 7], thin film deposits [8], dissolution of porous materials [9, 10], and dielectric breakdown [11]. A more extensive discussion of DLA and its relation to natural growth processes may be found in [12] and [13].

DLA clusters are known to be fractals. To illustrate this we show in figure 2 the result of a measurement of the average number of particles, $S(r)$, within a distance r of a given particle of a DLA cluster. To obtain $S(r)$, 100 particles p_i, $i = 1, \cdots, 100$ were randomly chosen from the first 2000 particles of the cluster shown in figure 1. When the cluster consisted of $s = 200\,000$ particles, the number of particles within distance r to particle p_i, $S_i(r)$, was counted for each of the selected particles. Finally, $S(r)$ was calculated as the average value of $S_i(r)$. For $l < r < r_{\max}$, the relation between $S(r)$ and r is consistent with the scaling law

$$S(r) \propto r^{D_f}. \tag{1}$$

Consequently, the particle density, $\rho \propto S(r)/r^d \propto r^{(D_f-d)}$, is not constant, but decreases when increasing the length scale on which the density is measured. This scale dependence of the mass density is apparent in figure 1, where larger clusters clearly are more sparse than smaller ones. The scaling exponent D_f can be interpreted as an effective fractal dimensionality of the cluster [14]. From figure 2 a fractal dimensionality $D_f \simeq 1.74$ may be read off, in reasonable agreement

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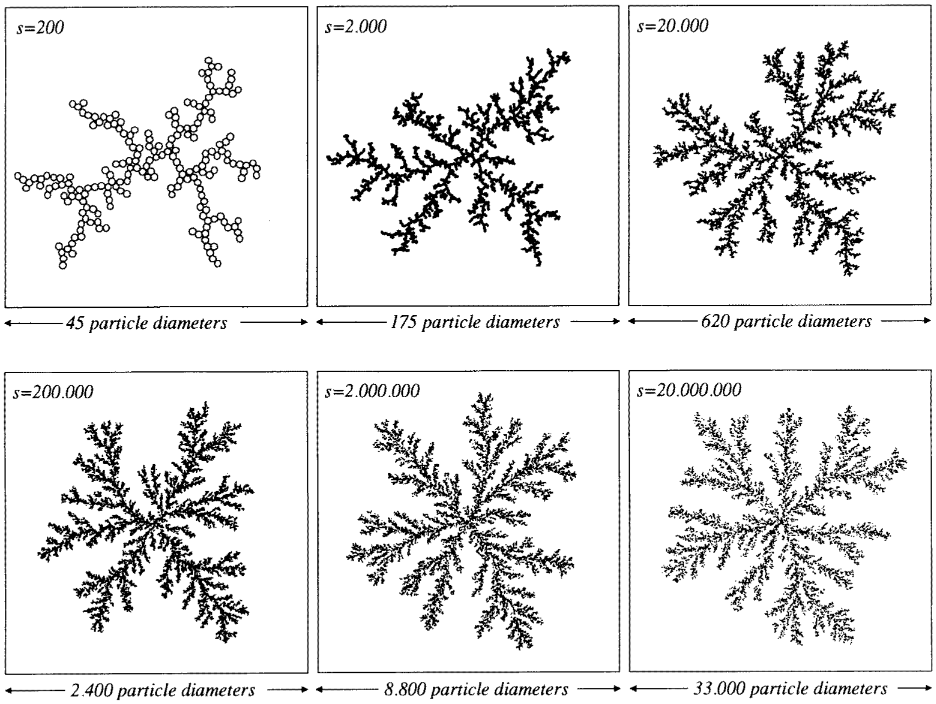

Figure 1. A two-dimensional, off-lattice DLA cluster at six different stages of growth. The cluster consists of identical discs of diameter l, and is shown at the stage when it contains s = 200, 2000, 20 000, 200 000, 2000 000 and s = 20 000 000 particles respectively. The cluster seed was one single disc. According to the finite printer resolution, only a fraction of all particles are shown for $s > 20\,000$ in order not to change the visual appearance of the cluster.

with the accepted value of $D_f \simeq 1.715$ for two-dimensional, off-lattice DLA [15], obtained from the dependence of the cluster radii of gyration on s. The two crossover lengths that define the regime in which the scaling law holds are the particle diameter, l, and the radius of the complete cluster of s particles, r_{max}, respectively.

2. Construction of a DLA cluster model

Studies of DLA are carried out through numerical simulations of the growth algorithm, such that the clusters usually ‘exist’ only in the memory of a computer. On occasion of the 50th birthday of Paul Meakin—a distinguished member of our research group who has contributed much to the present understanding of DLA—a macroscopic wood model of a three-dimensional DLA cluster was built. To our knowledge, this is the first time such an enterprise has been undertaken, and in the following we describe the construction process in detail.

Although the growth of DLA clusters is concentrated on their outmost tips, they have deep ‘fjords’ that can be penetrated by the random walkers during their diffusive motion [16]. As a result, there is always a chance that

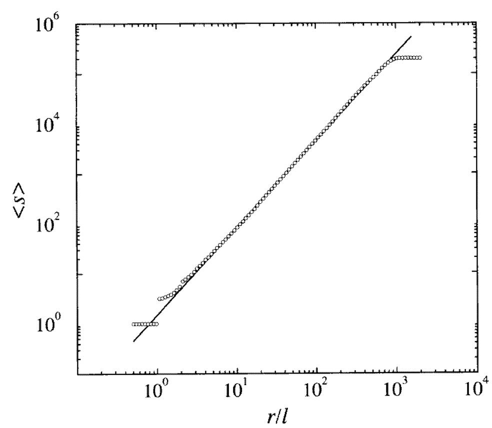

Figure 2. Log–log plot of average mass $S(r)$ within distance r to a given particle as a function of r for two-dimensional, off-lattice DLA. Filled circles are experimental data, obtained from the DLA cluster shown in figure 1 as explained in the text. The line corresponds to the fit $S(r/l) = 1.5\,(r/l)^{1.74}$, indicating a fractal dimensionality of $D_f = 1.74$ of the cluster.

a new particle can be added deep inside the existing

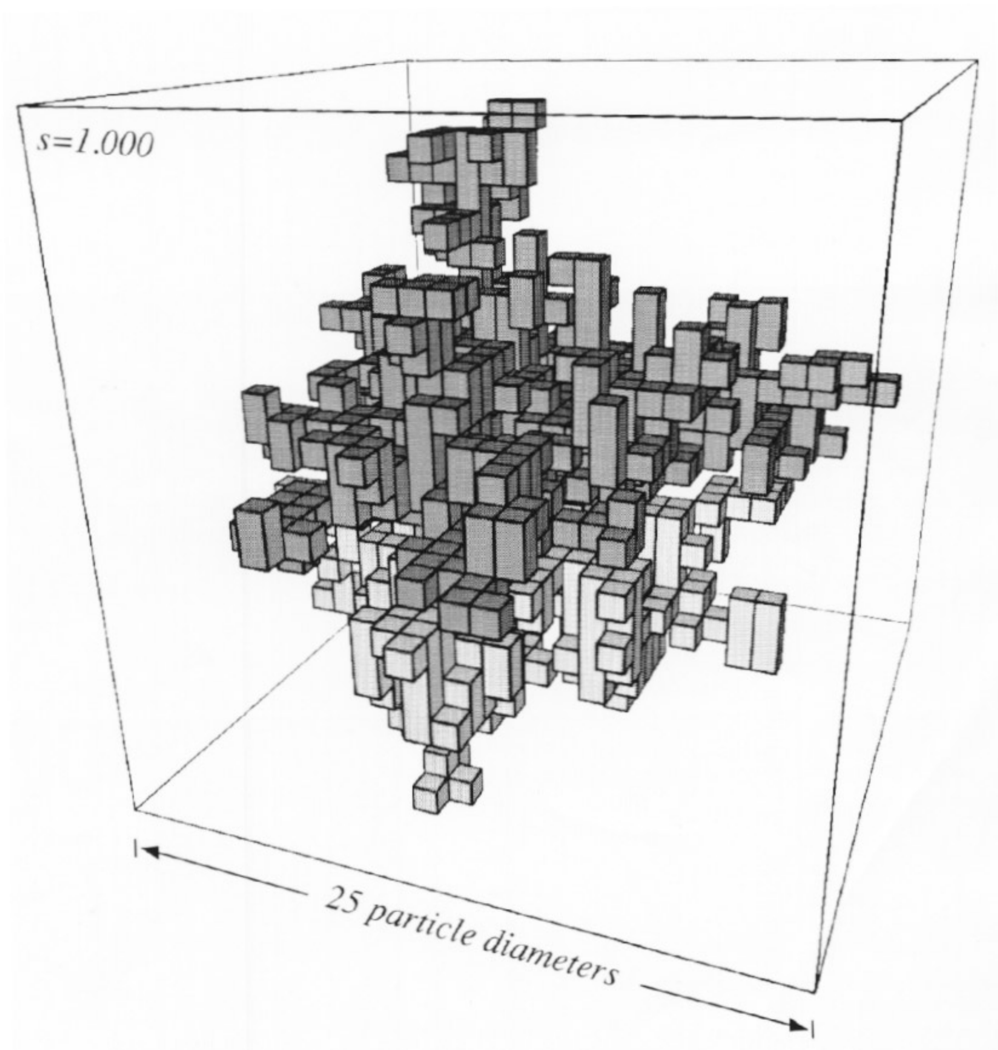

Figure 3. Perspective view of a three-dimensional, cubic lattice DLA cluster of mass $s = 1\,000$. The individual particles are identical cubes of side length l, but the cluster is displayed as if consisting of vertical columns of different lengths. The 'upper' and the 'lower' part are indicated (compare with the text).

cluster, at a position that is hard to reach from the outside. Consequently, it would be difficult to build the model of a three-dimensional DLA cluster in the way the cluster is grown, i.e. by adding one particle after the other. Instead, we first grew a cluster of a given mass on a computer, and then assembled the corresponding model from appropriate construction plans.

Very 'light' clusters (small s) do not display the characteristic fractal structure of DLA. On the other hand, the assembly of say a model with $s = 1000\,000$ particles would require an enormous amount of time. Additionally, even the largest cluster branches are usually attached to the cluster seed through only one single particle, such that very 'heavy' (large s) DLA clusters would be mechanically unstable. For our model the quite moderate mass $s = 1\,000$ was chosen.

The cluster was created using a lattice model in which the random walkers were confined to the sites of a cubic lattice with lattice constant l. The particles were identical cubes of size l, such that all particle–particle junctions were full side–side contacts. Figure 3 displays a perspective view picture of the cluster, in which each vertical row of i cluster particles is represented by a column of length $L = i\,l$. Using such columns instead of elementary cubes as the construction elements increases the stability of the model and makes its assembly easier, faster, and more precise. None of the individual columns cross the midplane that cuts the cluster into two parts of approximately equal size. The 'upper' and the 'lower' part are indicated in figure 3 by

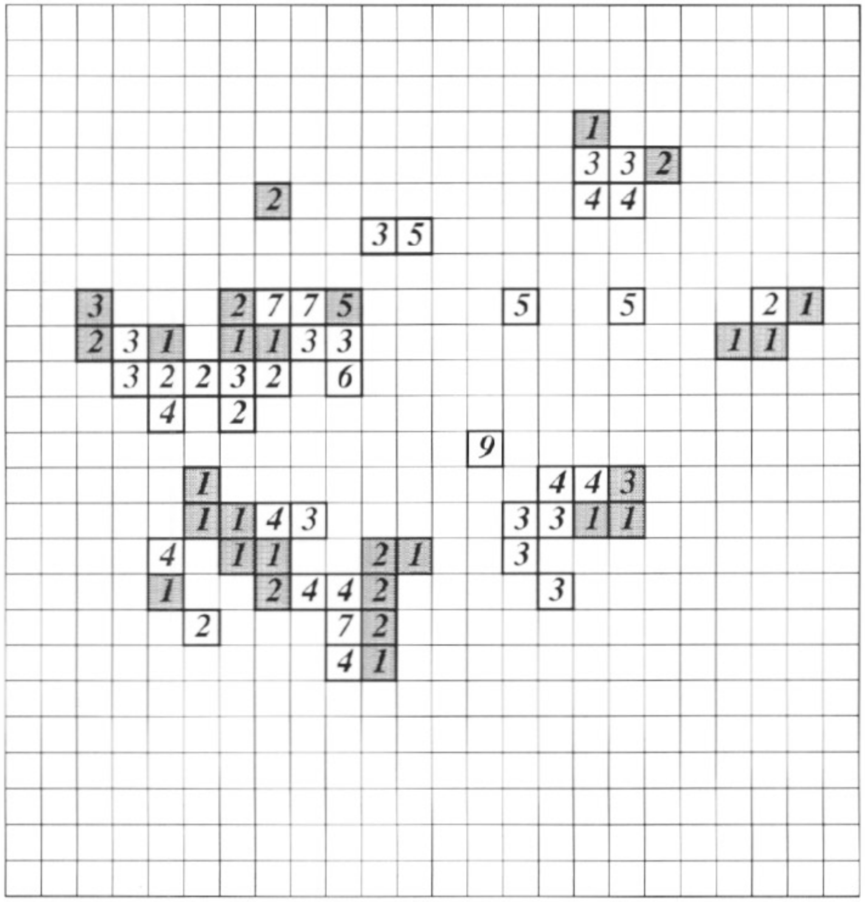

Figure 4. The blueprint of the cross section through the upper part of the DLA cluster in figure 3 to the midplane. The cluster model was assembled with the help of such blueprints.

using different colours for the columns that belong to different parts. When the cluster model was constructed, these two parts were assembled separately.

With a particle size of $l \simeq 1$ cm the model has a diameter of about 25 cm, and fits nicely on a bookshelf.

Construction plans for each of the two parts of the model were obtained from the positions of all the cluster particles. The complete plan for the upper part consisted of a number of blueprints of horizontal cross-sections $\mathcal{Z}_n$ through the cluster at different heights, $z_n = n\,l$, $n = 0, 1, 2, \ldots$, to the midplane. In each of the blueprints, all columns that passed through the corresponding section plane were represented as an outlined square, placed at the appropriate position. The length of the corresponding column was printed inside this square. Additionally, columns that cross $\mathcal{Z}_n$, but not $\mathcal{Z}_{n-1}$, were represented as shaded squares in order to improve the readability of the blueprints. As an example, the blueprint $\mathcal{Z}_4$ of the upper half of the cluster is reproduced in figure 4. The plan to construct the lower part of the cluster was obtained in a similar way.

On the basis of these blueprints, the entire hardware model of the DLA cluster was assembled in a five-hour session involving a team of six students. The building material was balsa wood. This type of wood is delivered in rods of various shapes and sizes. It combines extreme lightness with sufficient stiffness, is readily available, and cheap. The rods were cut into rectangular columns of the necessary lengths with a wood saw and a mitre box. Rough edges were smoothed using sandpaper.

The upper part of the cluster was assembled in the following way: all balsa columns emerging from the section plane $\mathcal{Z}_0$ were positioned on a sheet of paper according to the first blueprint. Neighbouring columns were glued together with superfast glue. Then, subsequent layers of the model were completed by putting all columns emerging from the corresponding section plane into place. Occasionally it was convenient to assemble small cluster branches separately before adding them to the model.

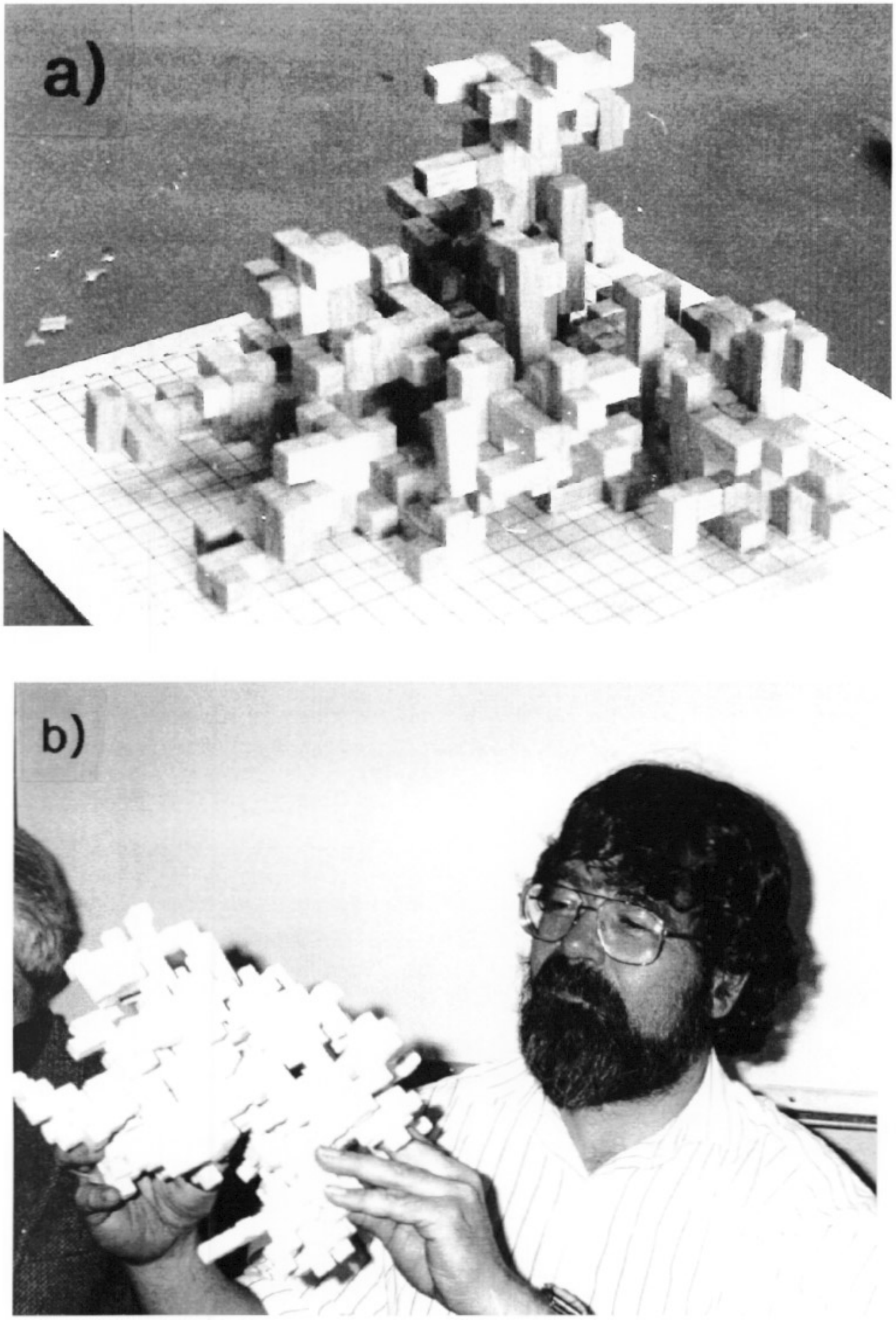

Figure 5. The DLA cluster model at (a) some intermediate state and (b) the final state of the assembly, when given to Paul Meakin.

When the upper part of the model was finished, it was turned upside down and the lower half assembled on top of the upper one. At this stage, the model had to be supported from below by hand. Figure 5 shows the cluster in an intermediate and in its final state, when it was given to Paul Meakin.

3. Conclusion

From our experience, constructing the model of a moderately sized, three-dimensional DLA cluster is possible as a classroom experiment. It visualizes the DLA algorithm in a way 'beyond' computer screens and creates a vivid impression of the intriguing structure of DLA clusters. It requires both craftsman's and computational skills and leads to intense social interaction among the students.

Our hardware model consists of balsa columns of different length, glued together with superglue. The cost of the material was less than $15. Apart from an ordinary saw and a mitre box, no tools were required. A FORTRAN-77 source code to generate three-dimensional DLA clusters is available from the authors upon request [17], but can be written by any student with some background in programming. With such a code, DLA clusters up to $s = 10\,000$ are easily generated on a PC.

The use of columns was time-saving, improved the precision of the assembly, and increased the stability of the model. Nevertheless, a great deal of concentration was required during the building process in order to keep track of the current construction stage, and to glue the balsa columns in a precise manner.

Acknowledgments

TR acknowledges financial support from the Norwegian Research Council. All authors would like to thank Paul Meakin for inspiring cooperation. We benefited from an introduction to DLA and many other concepts of modern statistical physics. Happy birthday, Paul!

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