Self-Organized Criticality: Consequences for Statistics and Predictability of Earthquakes

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The concept of self-organized criticality provides a natural, robust explanation of the statistics of earthquakes, including the Gutenberg-Richter law for the distribution of earthquake magnitudes. The dynamics is "at the edge of chaos" with algebraic, not exponential, divergence of small uncertainties. Temporal clustering of big earthquakes arises because of an underlying fractal structure of correlated regions. Scaling laws suggest that the statistics of large events can be inferred from the statistics of the much more numerous small events.

INTRODUCTION

The frequency of earthquakes versus magnitude exhibits a logarithmic dependence over many decades, known as the Gutenberg-Richter law [Gutenberg and Richter, 1956]. Figure 1 shows data for earthquakes in the New Madrid zone collected by Johnston and Nava [1985]. The magnitude is proportional to the logarithm of the energy release, or the seismic moment, so the G-R law indicates that these quantities obey a power law distribution, with exponent $1+\beta$, and $\beta \approx 0.8$. Measurements of the exponent β vary from place to place, so the exponent appears to be non-universal. The spatial distribution of earthquake occurrence seems to be fractal. The distribution of aftershocks follows Omori's law, another power law. Of course, such power laws can not extend to infinite energies; there has to be a cutoff somewhere: if for no other reason, then because of the finite size of the earth. From measurements over a finite period, say the last 60 years, all we can say is that this cut-off exceeds earthquakes of size 9.

These facts seem to imply that we are dealing with a kind of critical phenomenon, because power laws for temporal and spatial correlation functions are the hallmark of systems at a critical point for a continuous phase transition. Indeed, the mathematician Vere-Jones [1977] demonstrated that in principle a power law (with an exponent β=0.5) could be formally explained by thinking of an earthquake as a critical chain reaction, starting from a single rupture event. At each branching point the probability of doubling the activity must be precisely balanced by the probability of death of the activity (Figure 2). But why should the chain reaction be exactly critical, since with unit probability chain reactions are either sub-critical, in which case large events would be exponentially unlikely, or super-critical, in which case the activity would explode exponentially?

A few years ago one of the authors, in collaboration with Chao Tang and Kurt Wiesenfeld [Bak, Tang, and Wiesenfeld, 1987, 1988a,b; Wiesenfeld, Bak, and Tang, 1989; for a review see Bak and Chen, 1991] demonstrated that slowly driven dynamical systems, with many degrees of freedom, naturally self-organize into a critical state, with avalanches of all sizes obeying power law statistics. The critical state is an attractor for the dynamics. The phenomenon is deterministic and robust with respect to noise and inhomogeneities. Large catastrophic events are intrinsic, unavoidable properties of those large interactive systems, and no external or internal cataclysmic force is necessary. Thus, in contrast to equilibrium physics where criticality is the exceptional case, in non-equilibrium physics criticality could be the typical state of matter.

This opens up for an entirely new view on many sciences, including Biology [Raup, 1986], where catastrophic events have occurred intermittently in the past, most notable the extinction of the dinosaurs 50 million years ago, Economics, with power law tails for the fluctuations on Wall Street and the distribution of price variations [Mandelbrot, 1963], and Geophysics, not only in relation to earthquakes but also for volcanic eruptions [Diodati, Marchesoni, and Piazzo, 1991].

The obvious applicability of self-organized criticality to earthquakes was immediately and independently pointed out by several authors [Bak and Tang, 1989; Ito and Matsuzaki, 1989; Sornette and Sornette, 1989; Carlson and Langer, 1989]. The initial models were quite crude local sand-pile type models, but nevertheless served to illustrate the viability and the robustness of the idea. Subsequent rupture models [Chen, Bak, and Obukhov, 1991; Xu, Bergersen, and Chen, 1991] included the long-range

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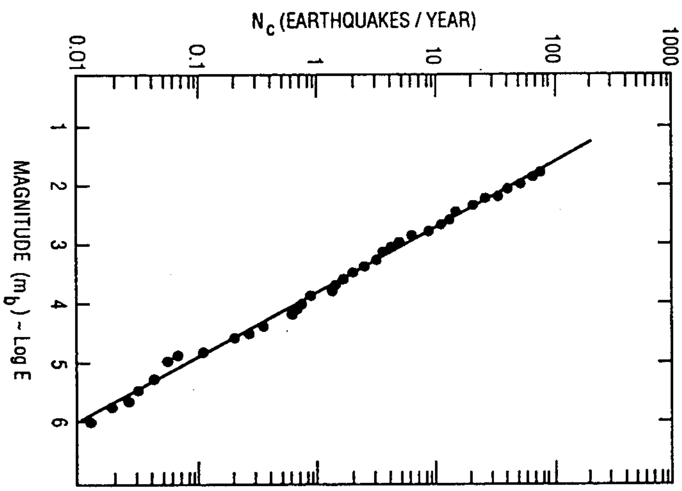

Fig. 1. Cumulative distribution of earthquakes in the New Madrid zone during the period 1974-1983. The data were collected by Johnston and Nava [1985].

redistribution of elastic forces following rupture. A thorough analysis of two-dimensional models derived from Burridge-Knopoff models [1967] of blocks connected with springs has been carried out by two of us, in collaboration with Hans Jacob Feder [Olami, Feder, and Christensen, 1992; Christensen and Olami, 1992a,b]. In contrast to the original models, these latter models do not invoke any conservation laws for force redistribution, thus removing one artificial artifact. The striking picture which emerges from the SOC theories is that the crust of the earth on which we are living operates at a perpetually critical state, always at the verge of collapse.

In the following we shall discuss results based on the study of SOC models of earthquakes, and point out the consequences for our ability of predicting earthquakes, in particular their statistical properties. Results for model calculations are discussed in the context of observations. Bear in mind that a more or less complete understanding of a physical phenomenon does not necessarily allow us to predict the future, as is the case in quantum mechanics, and for chaotic systems. Our motivation is a desire to understand, rather than to predict or prevent specific events. We find, however, that earthquakes is not a chaotic phenomenon, so at least there are no fundamental dynamical principles preventing us from predicting earthquakes.

We suggest that scaling laws be exploited to deduce the statistics for the few large earthquakes from the statistics of the much more numerous small earthquakes. Specifically, it will be argued that large earthquakes are clustered rather than periodic, contrary to popular belief (but in agreement with observations), and that the occurrence of large characteristic events are illusions based on some peculiar features of power law distributions. The danger of such mirages in fractal phenomena has been pointed out by Mandelbrot [1963, 1982].

SOC SPRING-BLOCK MODELS OF EARTHQUAKES

The idea of self-organized criticality, as applied to earthquakes, may be visualized as follows: Think of the crust of the earth as a collection of tectonic plates, being squeezed very, very slowly into each other. In the beginning of our geological history, maybe the stresses were small, and there would be no large ruptures or earthquakes. During millions of years, however, the system evolved into a stationary state where the build-up of stress is balanced in average by the release of stress during earthquakes. Because of the long evolutionary process, the crust has "learned", by suitably arranging the building blocks at hand into a very balanced network of faults, valleys, mountains, oceans and other geological structures, to respond critically to any initial rupture. The result of this self-organization process is in sharp contrast to any network of faults that one might set up by construction or engineering, which would certainly not be critical. We do not know how it all started, but that is not important for our arguments: the self-organized critical state is an attractor of the dynamics which will be reached eventually irrespectively of the initial conditions.

It makes no sense to separate the dynamics from the statics. It is not productive to think of earthquakes as being generated by "pre-existing faults". One can trivially explain the G-R law by assuming a fractal distribution of faults with a power-law distribution of characteristic fault sizes, but that leaves us with the equally difficult problem of explaining the dynamical origin of that distribution. Popularly phrased, one must take a holistic view of the situation. What appears to be a static configuration of large faults in a human lifetime merely constitute a snapshot of a slow ongoing geological process that has been hundreds of millions of years underway. During that period, faults have come and gone. The dynamics of the fault structure and the Gutenberg Richter law must be produced within a unified picture. The SOC models simulate the long term dynamics of the crust. In order to represent a realistic view of geophysics, the models must be robust, or adaptive, in the sense that if the physical properties were changed, or if noise were added, the system would reorganize during a transient period and become critical again. This is indeed the case for SOC models of earthquakes.

We want to study the simplest possible models which contain the essential physics of earthquakes. While there has been studies of three-dimensional crack-propagation models with slightly more realistic long-range redistribution of elastic forces following rupture, simple local models are probably more instructive, and certainly much more amenable to numerical and analytical study. We stress that we don't think of the Gutenberg-Richter law as originating from a single fault, which must necessarily have a characteristic energy depending on the size of that fault: our models

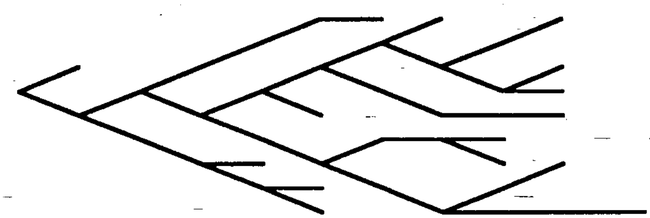

Fig. 2. Rooted tree generated by a critical branching process. Each branch indicates a rupture event, and the complete tree represents a single earthquake.

are "toy" models supposed to illustrate the principle of global organization of the crust of the earth.

Consider a two-dimensional lattice of interacting blocks. The initial block structure merely represents a discretization of the space in much the same way as the lattice in lattice gauge theories of particle physics. The block size does not represent an intrinsic length scale in the problem. On each block, at sites (i,j), acts a force $F_{i,j}$ in the general direction of motion. In the beginning $F_{i,j}$ may assume some random, small value. The initial state is not important for the long term dynamics. Let the force increase uniformly by a very small amount per unit time; this simulates the slow driving by the tectonic plate motion, or whatever force is driving the system. Eventually, the force at some site (i,j) must exceed a critical threshold value F_C for rupture. The critical force may either assume the same value at all sites, or be randomly distributed. The initial rupture is simulated by updating the forces at the critical site and the sites of the neighbors at (i,j±1) and (i±1,j). There are several possibilities for defining those rules:

a)

$$F_{i,j} \to F_{i,j} - F_C$$
$$F_{nn} \to F_{nn} + \alpha F_C$$

or b)

$$F_{i,j} \to 0$$
$$F_{nn} \to F_{nn} + \alpha F_C$$

or c)

$$F_{i,j} \to 0$$
$$F_{nn} \to F_{nn} + \alpha F_{i,j}$$

These equations represent the transfer of force to the neighbors. The updating of all sites is done in parallel. The transfer may cause the neighbors to be unstable and a chain reaction to take place. This chain reaction is the earthquake. The equations are completely deterministic. We are not dealing with a noise-driven phenomenon; on the contrary the physics turns out to be stable with respect to a small noise, i. e. noise is irrelevant. After a finite number of updatings, the forces on all sites will become sub-critical, and the earthquake stops. The system is then quiet until the force at some other location exceeds the critical value and a new event is initiated. The process is repeated again and again. One observes that the earthquakes become bigger and bigger for a long transient period. Eventually the growth stops: the crust has self-organized into a stationary state. At this point one may start measuring the seismic moment of earthquakes as defined by the total number of rupture events following a single initial rupture. A histogram similar to that in figure 1 for real earthquakes can be constructed.

The difference between the definitions a), b) and c) may seem subtle and irrelevant since one might expect $F_{i,j}$ to be not far from F_C at rupture, and certainly, by definition, is identical to F_C at the initial triggering instability. Actually, this difference turns out to be essential. In the continuous version of the BTW model for self-organized criticality (Bak, Tang, and Wiesenfeld, 1988b; Bak and Chen, 1991a) the equations a) were applied, with α=1/4, so the force was conserved. However, as soon as α deviates from 1/4, the criticality gradually disappears, with a decreasing cut-off for large earthquakes. It turns out that there is no reason that the force be conserved for real earthquakes, so the model is not robust enough.

Recently, it was accidentally discovered by Feder and Feder [1991] that the situation b) with α = 1/4 allowed for some criticality. Note that this model is non-conservative because sometimes the value at the critical site is larger than the threshold value F_C so the reduction of force at that site exceeds the amount transferred to the neighbors. Olami, Christensen and Feder realized that model c) could be directly related to earlier spring models studied for example by Burridge and Knopoff [1967], with the value of α directly related to the elastic parameters. The criticality in this case prevails for values of α down to 0.05, with only 20% conservation. This came as a surprise since there was at that time a widespread belief that the lack of conservation would spontaneously generate a length scale, i. e. a "characteristic earthquake size". It turns out that criticality occurs generically: it is almost independent of the details of the toppling rule. In fact, the situation a) is the one which is special in the sense that its rules do not induce correlation between toppling sites.

Figure 3 shows the distribution of earthquakes for α = 0.20. The straight line on the log-log plot indicates a power law: the system has self-organized into the critical state. The slope of the line corresponds to a β value of 0.8. The slope turns out to depend on the degree of dissipation, (1-4α), so there is no universality of the exponent β in the non-conservative case. One should not look for unique b-values in nature. Indeed different b-values are observed in different regions of the world.

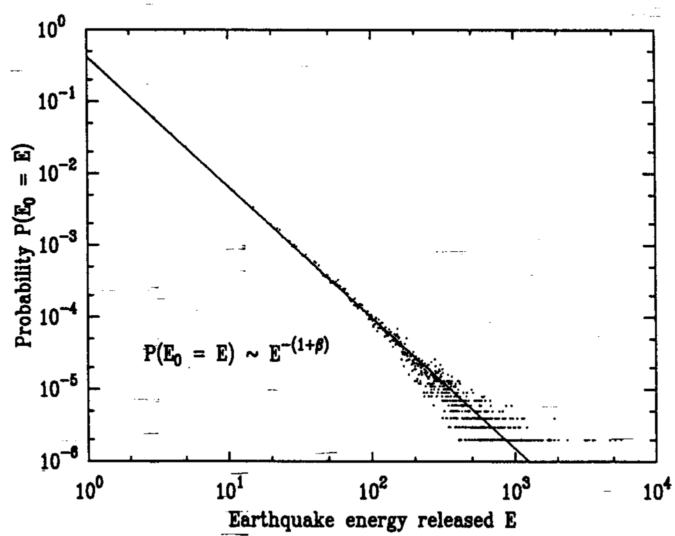

Fig. 3. The distribution function for the energy of earthquakes in our model. This graph represent a total of half a million earthquakes on a square lattice of linear size 100, with α=0.20.

Note the single scattered events for values of E greater than the one where n(E)=1, i. e. the point where the straight line crosses the line n=1 ($2x10^{-6}$ in the plot). This is due to the fact that the power law distribution function has the peculiar property that the average size of earthquakes diverges, so that a sampling from the distribution function never converges to the distribution function itself. The difference amounts to almost one full order of magnitude, and can not, even in principle, be eliminated by better statistics which would simply shift the problem to larger earthquakes. We are condemned to accept the fact that the statistics is poor precisely for the events that we are most interested in, namely the large events responsible for the highest energy release, and the most damage.

The observation of such single, seemingly atypically large earthquakes, has led to the concept of "characteristic earthquakes" not given by the G-R distribution (see Scholz, 1991) The size of these characteristic earthquakes is merely a consequence of the finite duration of the observation, typically something like a human lifetime. The slow logarithmic dependence of the "typical largest earthquake" on the observation period might lead to the belief that the time-scale, and the magnitude of those events are significant. Common sense, however, indicates that a human life time can play no role in a geophysical phenomenon such as earthquakes. Were we to live a million years, we would probably observe "characteristic" earthquakes of magnitude 13. Kagan [1992] has pointed out from analyzing actual earthquake catalogs that characteristic earthquakes indeed seem to be statistical artifacts. We shall see that those observations may actually be related to earthquake clustering.

Figure 4 illustrates the slow nature of the self-organization process. The running average of earthquake sizes vs. time is shown

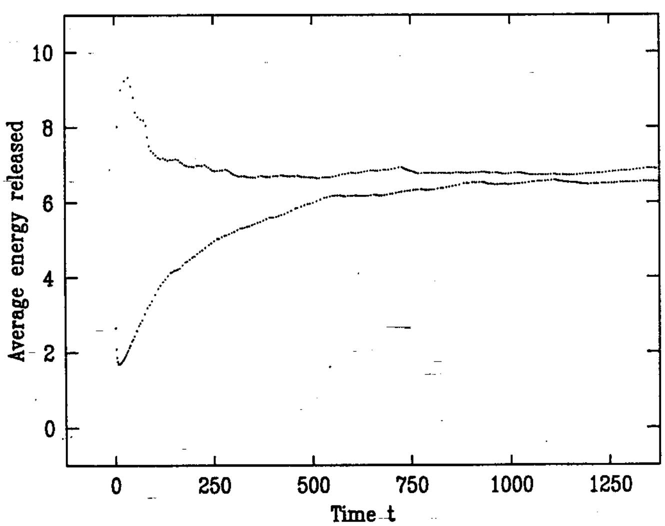

Fig. 4. Average size of earthquakes as a function of time during the self-organization process for a system of size L=70, conservation level α=0.20, The lower graph represents the slow growth of the average for an initial random lattice. The growth rate is a measure of the correlation length in the system. Notice that the initial rise is linear. The saturation is an effect of the finite system size. The upper graph is data for an initially correlated lattice.

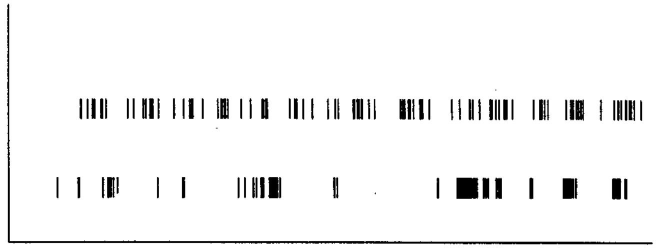

Fig. 5. A sequence of earthquakes for a 35x35 system with α=0.20. The upper sequence shown is the occurrence of earthquakes with energy greater than or equal to 20. The lower sequence is for the occurrence of earthquakes with energy larger than 450. The time intervals are scaled so that the densities of events are the same. Note the clusters of "characteristic earthquakes". The life-time of the clusters may represent the active lifetime of an individual fault or fault structure.

in the lower curve, starting from a random uncorrelated configuration of forces. The growth will continue indefinitely, limited only by the size of the system. During the self organization process a time dependent cutoff will be seen in the size of the earthquake distribution. The upper curve shows the running average starting from a state which has already had time to reach the steady state. The initial fluctuations in the curves can be shown to be due to the fact that the exponent of the power law distribution is less then 2.

When comparing with real earthquake statistics, we assume that the crust of the earth has had sufficient time to complete the self-organization process. The power law distribution of earthquakes stems from the fractal nature of the SOC state, with correlated regions ranging over all length scales; those correlated regions, generated by the long term dynamics, are the equivalent of the active faults, or fault segments, in real earthquakes. The fault structure changes on large geological time-scales. The long range SOC models (Chen et al [1989] and Xu et al [1991]) produce a geometry which looks much more like a real fractal-like arrangement of two-dimensional faults in a three dimensional matrix.

Temporal Correlations

A very intriguing question of earthquakes is the temporal correlations between earthquakes. In figure 5 we show two temporal sequences of earthquakes derived from model c) with α=0.20 for earthquakes with energy larger than 20 and 450, respectively, for a 35x35 system. It is evident that the two time sequences are dramatically different. The sequence for small earthquakes seems to be random, while for the large earthquakes the distribution is highly clustered. Also, the centers of the clustered earthquakes are generally correlated in space.

A possible measure for the observed temporal clustering is the coefficient of variation, $C_V(E)$. It is defined as the ratio between the square root of the variance of the temporal intervals, $Var(t)_E$, and the average interval between earthquakes with energy larger than E, $\langle t \rangle_E$. For a random signal the distribution function is simply an exponential function yielding $C_V(E) = 1$. For a periodic signal $C_V(E) = 0$ while clustered earthquakes will produce $C_V(E) >$

1. We have measured the coefficient of variation for the conservative model with α=0.25 and for a non-conservative model with α=0.20. No correlations are seen between earthquakes in the conservative model, $C_V(E) \approx 1$. In the non-conservative model we see a clustering effect for large earthquakes, see figure 6, while random behavior is observed for small earthquakes in accordance with figure 5. The decrease in the coefficient of variation for very large earthquakes is a finite-size effect, related to the cutoff in the energy distribution. The same kind of temporal correlations is seen in real earthquakes. Small earthquakes seem to be uncorrelated [see figure 6 in Johnston and Nava, 1985]. Large earthquakes display strong clustering [Kagan and Jackson, 1991]. If one were to perform the same types of calculation for small subsystems, correlations between smaller events should eventually appear. This implies that it might be very useful to study smaller events in order to get more understanding of the larger events: one might predict the statistics of large earthquakes in large regions by scaling properly the statistics of smaller earthquakes in small regions.

The self-organization process of earthquakes creates big correlated strain structures which are responsible for the occurrence of large earthquakes. Usually those correlated structures do not disappear after a shock has occurred. Because of strain dissipation during the shock the strain in a correlated areas drops to a lower value. The same correlated areas have a relatively large probability to be activated again after a short loading time which is defined by the degree of non-conservation. The system remembers its past. This is the basic explanation for the spatio-temperal clustering of large earthquakes. It should be noted that the average time interval between large earthquakes is very large (proportional to E^{β}) while the characteristic time between clustered earthquakes is simply the loading time of the strain. Those two time scales are very different.

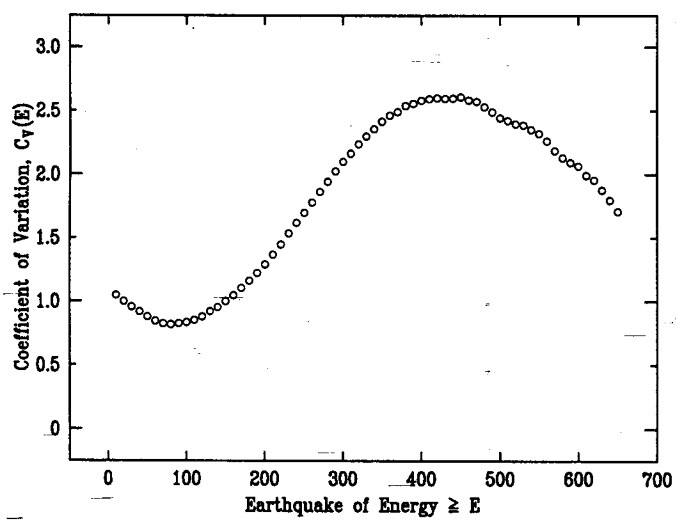

Fig. 6. The coefficient of variation $C_V(E)$ as a function of the energy released during an earthquake. The results are for α=0.20, L=35 and open boundary conditions. For the smaller earthquakes $C_V(E)<1$. The larger earthquakes are characterized by clustering, $C_V(E)>1$.

Thus, *locally* one observes a clustering of earthquakes of a certain magnitude related to a "pre-existing" correlated region, which might take the shape of a "fault". The system's memory is encoded into the fault structure. However, the cluster survives only for a certain time, i. e. the time scale for which the fault remains active; that is the interval between clusters in figure 5; this is long compared with the interval between large earthquakes, but short on a geological time scale. If one averages over long time, or over a large enough geographical area, the correlations disappear and the Gutenberg-Richter law is recovered. This was actually observed by Scholz in his study of earthquakes in Alaska (Scholz, 1991): data from individual "rupture zones" appear to include characteristic earthquakes, but when averaged over the whole zone the earthquakes obeyed the Gutenberg-Richter law.

ARE EARTHQUAKES CHAOTIC?

There has been a good deal of speculations that many complex phenomena in Nature are chaotic. In chaotic systems, a small uncertainty δ of the initial state of the system grows exponentially with time, $\delta = e^{+\lambda t}$ where λ is known as the Lyapunov exponent. This exponential growth makes the behavior of the system unpredictable at times larger than a characteristic time $1/\lambda$. But critical systems have no characteristic time scale so how can they be chaotic?

Within the simple uncorrelated chain reaction picture the question has a simple answer. Compare the system with another critical system which initially is slightly different, for instance by starting with slightly different values of the forces. This causes a number of "mistakes" in the branching process illustrated in figure 2. Some sites which branch in the original model die, and vice versa. The number of mistakes grows linearly in time. The difference in the state of the two systems is simply the accumulated number of mistakes, which grows quadratically with time, $\delta = at^2$, not exponentially. In the more general case, where the branchings are correlated, one expects the divergence to be given by a different power law. Indeed, this is what is found in numerical studies of earthquake models [Bak and Chen, 1992; Chen, Bak, and Obukhov, 1991] and other self organized critical phenomena. Our conclusion based on the criticality indicated by the Gutenberg-Richter law and numerical simulations must be that earthquakes are not chaotic, so the structure of the dynamical equations does not in itself prevent earthquake prediction. Actually, the concept of self-organized criticality complements the concept of chaos wherein simple systems with a small number of degrees of freedom can display quite complex behavior.

The idea of self-organized criticality applies not only to earthquakes, but probably to most phenomena in Geophysics where power laws such as those characterizing spatial fractality have been observed. In particular, the intermittent nature of volcanic eruptions share many of the statistical features of earthquakes. Very recently Diodati et al., [1991] analyzed emission from the Italian volcano Stromboli and argued that it is indeed a self-organized critical phenomenon.

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