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Temporal correlations, universality, and multifractality in a spring-block model of earthquakes

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We characterize the complex temporal structure of the earthquake activity in a nonconservative spring-block model of earthquakes. The temporal sequence of the occurrence of earthquakes is observed to be multifractal. The clustering (temporal correlations) between earthquakes is characterized by a universal exponent. Those correlations disappear when the model becomes conservative.

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The internal organization of driven nonequilibrium systems has received a lot of attention recently. The sandpile cellular automaton suggested by Bak, Tang, and Wiesenfeld (BTW) is an example of such a system [1]. BTW showed that it is representative of a certain class of conservative systems which drive themselves into a statistically stationary state, characterized by power-law spatial and temporal distribution functions. Hence, the systems have no intrinsic length or time scales and is in this sense critical. This type of behavior was named self-organized criticality (SOC).

The seismic system is an example of a physical system displaying power-law behavior. The power-law distribution for the earthquake intensity is interpreted as a signature of SOC: The movement of the tectonic plates drives the system into a critical state [2].

We have recently proposed a continuous, nonconservative cellular automaton to describe the dynamics of a driven spring-block system modeling earthquakes. This nonconservative model displays robust SOC over most of its parameter range. The critical exponents are not universal, but depend on the degree of the nonconservation and on the boundary conditions as presented in detail elsewhere [3].

The temporal behavior of this model is interesting because of several reasons. First, it is known that earthquakes display fractal clustering and correlated behavior (see [4] and references therein). For shallow earthquakes, the amount of clustering after large earthquakes is described by a universal power law: the Omori law. We have studied the temporal behavior of this dynamical model and shown that it displays clustering for the larger earthquakes. Second, since the model displays nonuniversality for the critical parameters related to the earthquakes, such as the b value of the Gutenberg-Richter law, the question is whether there exists any universality for other critical exponents associated with temporal clustering. Finally, another interesting issue is the difference between the conservative model and the nonconservative model. There is a discontinuous transition of the scaling exponents when going from a conservative into a nonconservative system. However, there seems to be more fundamental changes in the temporal behavior, which are related to the fact that SOC in a nonconservative model can occur only if there are correlations between sites in avalanche clusters. The relative stability of those clusters are responsible for the observed clustering.

In this paper we focus on the temporal set of earthquakes which are generated by the deterministic dynamics of the spring-block model. We calculate the fractal dimension of the temporal sequence of earthquakes and find that it displays a multifractal behavior. We calculate the earthquake correlation function and show that it displays a universal behavior for critical nonconservative systems. The observed exponent is that reported in the Omori law. The total clustering associated with an earthquake depends on the size of the earthquake, the system size, and the amount of conservation. We characterize the scaling of this function. A system-size-dependent cutoff time appears in the problem; above this time no correlation is seen. We interpret those results as related to memory effects which are essential if any criticality is to be seen in such a system.

We first describe our model briefly. It can be derived from a two-dimensional spring block model for earthquakes [3]. It is defined on a two-dimensional lattice by a set of dynamical variables $F_{i,j}$ that represent the local force on a site (i,j), where $1 \leq i,j \leq L$. The local forces increase uniformly at a very slow rate until one site reaches a threshold value F_{th}. Then the forces on the unstable site and its nearest neighbors are updated according to the simple relaxation rule

$$
\begin{aligned}
&F_{\text{NN}} \to F_{\text{NN}} + \alpha F_{i,j} , \\
&F_{i,j} \to 0 .
\end{aligned}
\tag{1}
$$

This initiates an avalanche which either stops immediately or propagates further in the system. The distribution function of the size of the avalanches is a power-law distribution, where the cutoff scales with system size L, see Ref. [3]. The parameter α is related to the spring constants of the spring model. Notice that this system is nonconservative if $\alpha < 0.25$.

We can characterize the occurrence of each avalanche by an occurrence time defined by the driving of the system (the increase of strain is proportional to the elapsed time).

We discuss the earthquake temporal sequence $\mathcal{S} = \{E_{t_i}\}$, where t_i is the occurrence time of the earthquake and E is its energy. Subsets $\mathcal{S}_{E_0} = \{E_{t_i} \in \mathcal{S} | E \geq E_0\}$ of this temporal sequence are defined by assigning a minimal energy

E_0 for the earthquake under consideration, see Fig. 1.

We can ascribe a fractal dimension $D(E_0)$ to each subset $\mathcal{S}_{E_0}$ (see [5] for a detailed definition). We measure the number of intervals $N(\delta,E_0)$ of size δ needed to cover the subset $\mathcal{S}_{E_0}$, and we find

$$N(\delta,E_0) \sim \delta^{-D(E_0)} . \tag{2}$$

The fractal dimension $D(E_0)$ depends on the energy E_0. Figure 2(a) is the result of counting intervals for a system of size $L=70$ for $\alpha=0.20$ for three different energies $E_0=690$, 1608, and 2297. Notice that the interval determining the fractal dimension grows with the minimal energy E_0. When the size of the system is enlarged the intervals belonging to the scaled avalanches grow like L^{γ}, $\gamma=0.45$–0.64.

Figure 2(b) displays the measured fractal dimension as a function of the minimal energy E_0. The fractal dimension for small earthquakes is 1, which is the characteristic of a random set. The results in Fig. 2 are given only for energies where the interval is big enough to distinguish the fractal exponent. A nontrivial fractal dimension appears for larger earthquakes, indicating clustering.

In order to characterize the clustering more precisely we define an earthquake clustering function in the following way:

$$g(t)=\langle n(t)\rangle_{t_i} - t\bar{n} , \tag{3}$$

where $\langle n(t)\rangle_{t_i}$ is the number of avalanches in the interval (t_i,t_i+t) averaged over all t_i in $\mathcal{S}$, and $\bar{n}$ is the average density of earthquakes. $g(t)$ is simply related to an integral of a coarse-grained correlation function. For a Poisson process $g(t)$ is identically zero. We define $g_{E_0}(t)$ as $g(t)$ calculated for the subset $\mathcal{S}_{E_0}$, i.e., we restrict ourselves to the earthquakes with an energy release larger than E_0. The excess number of earthquakes a time t after the avalanche is $dg(t)/dt$. It is also proportional to the temporal point-point correlation function. For large earthquakes this behavior was reported to be a $1/t$ behavior.

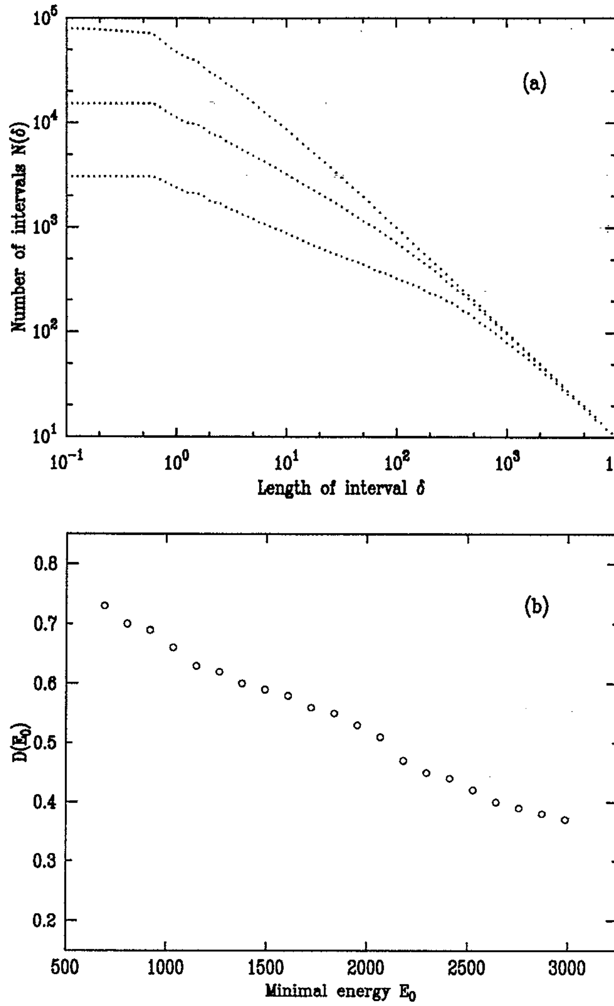

FIG. 2. (a) The number of intervals needed to cover the subset $\mathcal{S}_{E_0}$ for $E_0=690$, 1608, and 2297. Notice the existence of a minimal temporal distance between two successive earthquakes of a given minimal energy E_0. The range determining the fractal dimension grows with system size as well as with the minimal energy E_0. (b) The fractal dimension $D(E_0)$ of $\mathcal{S}_{E_0}$ as a function of the minimal energy E_0.

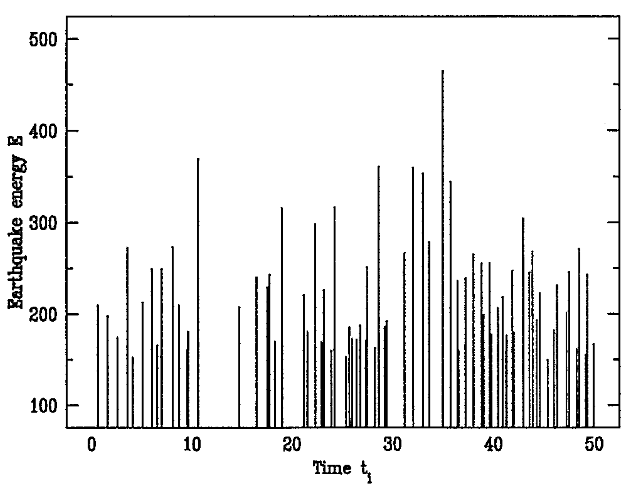

FIG. 1. A subsequence $\mathcal{S}_{150}$ of earthquakes. An earthquake with energy release E occurring at time t_i is represented by a δ function of height E.

To measure this correlation we generated large data sets. Since we expect to see different clustering for large and small earthquakes we measured the functions $g_{E_0}(t)$ for the full range of energy. We give an example for the results in Fig. 3. The basic form of $g_{E_0}(t)$ for the long-term behavior is

$$g_{E_0}(t) \sim \begin{cases} \ln(t), & t < t_{co}(L), \\ f(E_0,L), & t > t_{co}(L) \end{cases} . \tag{4}$$

The basic logarithmic dependence does not depend on α in the nonconservative critical models. This behavior seems to be *universal*. The exponent of the Omori law will be the same for all nonconservative models, namely, -1.0. The temporal correlations of earthquakes in this model display universality even though the exponents for the frequency-energy distribution display no universality. The cutoff time $t_{co}(L)$ for the logarithmic behavior does

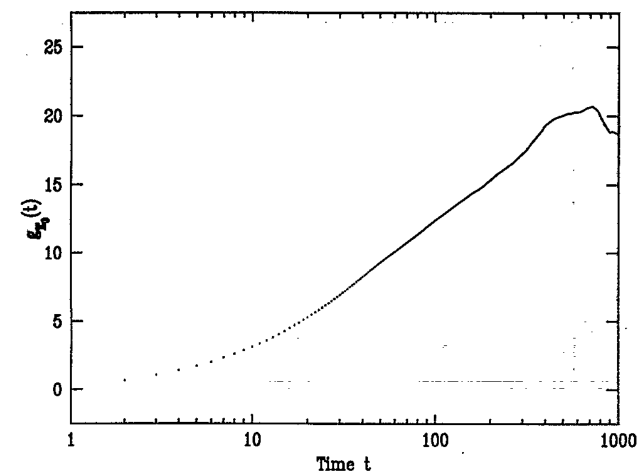

FIG. 3. The earthquake clustering function $g_{1608}(t)$ for a system of size $L=70$ with $\alpha=0.20$.

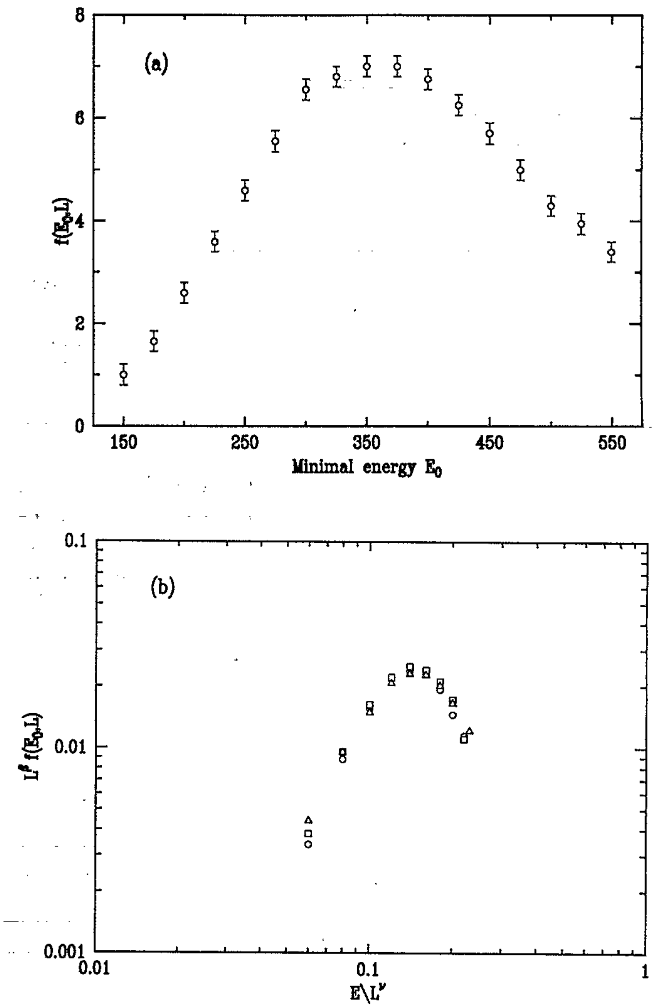

FIG 4. (a) Simulation results for $f(E_0,L)$ in a system with $\alpha=0.20$ and $L=35$. (b) Finite-size scaling plot of $f(E_0,L)$ for $\alpha=0.20$ and $L=35$ (circles), 50 (squares), and 70 (triangles). The scaling indices are $\nu=2.2$ and $\beta=-1.6$.

not depend on the minimal energy of the set, but it depends on α and the system size L. We present results for $f(E_0,L)$ for $\alpha=0.20$ and $L=35$ in Fig. 4. Below a minimal energy too much fluctuations are seen to distinguish any correlation. Notice that the drop at larger energies is a result of the fact that they are near the cutoff. We study the scaling of $f(E_0,L)$ with the system size L. The finite-size scaling hypothesis

$$f(E_0,L)=L^{-\beta}g(E/L^{\nu}) \tag{5}$$

seems to work well with $\nu=2.2$, which is the scaling exponent reported in [3], and $\beta=-1.6$.

We further checked the dependence of the cutoff time t_{co} on α. For $\alpha=0.25$ the cutoff time is zero, as the model becomes nonconservative a finite cutoff time appears. The cutoff increases with decreasing α. For $\alpha=0.20$ we checked the dependence of t_{co} on system size L, and it grows algebraically with L: $t_{co}(L)\sim L^{0.7}$. Thus, for $\alpha<0.25$ this cutoff time will diverge with system size. Notice that the exponents describing the growth with system size of the scaling regions of phenomena related to the earthquake temporal sequence are small.

The conservative case is characterized by a complete lack of temporal correlations. This is a result of the conservative nature of the model. The avalanches are very big and multiple relaxations can occur at sites in the system during an avalanche. Since the avalanches are conservative the interoccurrence time is much smaller. Both effects drive the correlation time to zero. However, for nonconservative models the situation is very different. Since the model is nonconservative no multiple relaxations will occur during an avalanche. Moreover, such models can become critical only if internal correlations are created inside the lattice. Correlated clusters are created by the avalanches. On the other hand, those clusters will be modified by avalanches but not completely destroyed. This is the reason for the increase in the cutoff time as well as for the "memory" effects in the avalanches.

One should notice that the clustering observed in this model is for time scales which are larger than the buildup time of an earthquake. Thus the clustering we observe is a long-term clustering. The exact relationship between this and the Omori law for aftershocks (which might be related to instabilities generated by a large earthquake) is somewhat unclear. Similar calculations on earthquake data were done by Kagan and Jackson [4]. However, it is very hard to deduce any exact exponents from their data.

We have shown that nonconservative models for earthquakes display a complex multifractal behavior. The nonconservation imply the existence of strong correlations between earthquakes. The exponents governing this clustering are universal for nonconservative critical systems. Furthermore, the total clustering is related to the earthquake energy through the function $f(E_0,L)$. The transition to a conservative model is associated with a zero correlation time.

Other temporal measurements like point-point correlation function and probability distribution for interoccurrence times do not display universality.

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