

**Christensen Replies:** Based on simulations with lattice sizes  $L=25, 35$ , and  $50$  it was shown that the cutoff in energy distribution  $E_c$  scales with system size  $L$  like  $E_c \propto L^{2.2 \pm 0.1}$  when  $\alpha=0.20$  [1,2]. I agree that the scaling  $L^{2.2}$  of  $E_c$  with system size  $L$  cannot be maintained for very large system sizes as pointed out by Klein and Rundle [3].

Klein and Rundle show this by assuming that (a) a slip dissipates at least an amount of  $(1-4\alpha)F_{th}$ , (b) the maximal stress in a 2D system of size  $L$  has to be less than  $F_{th}L^2$ , and (c) the total dissipation in an earthquake has to be less than the maximal stress in the system, i.e.,

$$E_c(1-4\alpha)F_{th} \leq F_{th}L^2. \quad (1)$$

This implies that  $E_c$  cannot scale with a system size like  $L^{2.2}$  for large systems.

When estimating the system size at which the contradiction becomes obvious Klein and Rundle use  $E_c = L^{2.2}$ , thus predicting a breakdown above  $L=3125$ . However, in this calculation one has to raise a constant to the fifth power, so the estimate of this constant is crucial:  $E_c$  scales like  $L^{2.2}$ , but the actual size of the cutoff energy is  $E_c \approx 0.18L^{2.2}$ . Furthermore, simulations show that the average stress in the system (when  $\alpha=0.20$ ) is well approximated by  $0.54F_{th}L^2$ . This is a more appropriate measure for the upper bound of stress in the system. Thus the criterion that the dissipation in an earthquake with the size  $E_c$  should be less than the (average) stress in the system reads  $0.18L^{2.2}(1-4\alpha)F_{th} \leq 0.54F_{th}L^2$  and the contradiction becomes obvious when  $L \approx 76000$ .

There is, however, an additional argument why the scaling of the cutoff energy cannot be like  $L^{2.2}$  and this sets in before that given by Klein and Rundle. When  $\alpha=0.20$  blocks slip only once during an earthquake. This can be verified numerically, but is rather obvious: Suppose an earthquake is initiated at a site  $i$ , i.e.,  $F_i = F_{th}$ , and that the four nearest-neighboring sites *all* have a stress  $F_{nn} = F_{th} - \epsilon$  close to the threshold value  $F_{th}$ . After the slip of the central site  $i$ ,  $F_i = 0$  and  $F_{nn} = F_{th} - \epsilon + \alpha F_{th} > F_{th}$ . Thus the neighboring sites all have to relax, and after this relaxation the stress at site  $i$  becomes  $F_i = 4\alpha(F_{th} - \epsilon + \alpha F_{th}) < F_{th}$ , when  $\alpha=0.20$ , i.e., the initiating block—the prime candidate to slip more than once—slips only once. (In order to see multiple topplings one would need  $\alpha \geq 0.21$ .)

Since the blocks slip only once the size of the cutoff has to be less than the number of blocks in the system, i.e.,  $E_c \approx 0.18L^{2.2} \leq L^2$  which is inconsistent above  $L \approx 5300$ . This restraint sets in before that given by Klein and Rundle, thus being the “true” origin for the

breakdown of the  $L^{2.2}$  scaling.

However, the primary question is *not* for which lattice size the breakdown of the scaling takes place, but *if it will affect the main conclusion that the nonconservative spring-block models defined in Ref. [1] display criticality with a power-law exponent depending on the amount of conservation*.

Below I list some arguments supporting the statement that the breakdown of the scaling will *not* affect the nonuniversal critical behavior of the system.

(1) The observation that the cutoff scales like  $L^{2.2}$  for small system sizes when  $\alpha=0.20$  indicates that the critical organization of the system will occur in larger systems as well. Indeed, simulations by Grassberger [4] show that self-organization occurs up to at least  $L=1000$  (the largest system size simulated so far for this model).

(2) The number of earthquakes in the model with energy  $E$  follows a power-law distribution. With  $\alpha=0.20$  we have  $P(E) \propto E^{-1.9}$ , i.e.,  $P(E_c) \propto L^{-4/2}$ . Thus the number of earthquakes with a size comparable to the cutoff energy is extremely small in a large system, and they are not likely to change the overall behavior.

(3) In Ref. [2] it was found that when  $\alpha \geq 0.22$ ,  $E_c \propto L^\nu$  with  $\nu \leq 2$  which does not violate the constraints in Eq. (1). The small system sizes used are large enough to see the “true” scaling behavior. It is important to note that in the range  $0.22 < \alpha \leq 0.25$  the nonconservative spring-block model displays criticality where the power-law exponent changes continuously from approximately  $-1.65$  to  $-1.22$ .

Thus I strongly believe that the conclusions about the critical behavior in nonconservative systems are valid, in spite of the fact that when  $\alpha \leq 0.20$  we have not yet reached the true scaling region.

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