Surface Fluctuations and Correlations in a Pile of Rice

A. Malthe-Sørenssen,[1] J. Feder,[1] K. Christensen,[2,*] V. Frette,[3] and T. Jøssang[1]
[1]*Department of Physics, University of Oslo, Box 1048 Blindern, N-0316 Oslo, Norway*
[2]*Department of Mathematics, Imperial College, 180 Queen's Gate, London SW7 2BZ, United Kingdom*
[3]*Institut für Experimentelle Physik, Otto-von-Guericke-Universität, Postfach 4120, D-39016 Magdeburg, Germany*
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The surface geometry and dynamics in a one-dimensional, slowly driven pile of rice are studied for two types of rice, a long-grained rice that displays power-law distributed avalanches and a rounder rice with a characteristic avalanche size. The surface fluctuations are proportional to a power of the system size with exponents $\chi = 0.2 \pm 0.1$, for both types of rice. The time correlation functions for the mass and height of the pile are characterized by a crossover time that is proportional to a power of the system size. The scaling behavior is similar for both types of rice, but the correlation time is much larger for the long-grained rice.

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Piles of sand have become the canonical example of how slowly driven, dissipative systems might evolve into a stationary state characterized by power-law probabilities for various quantities. Bak *et al.* [1] showed that this idea of self-organized criticality (SOC) applied to a simple model of a slowly driven pile of sand. Models based on similar ideas have been used to explain the occurrence of power-law fluctuations in systems as diverse as earthquakes [2], front dynamics [3]. and evolution [4]. However, the question of whether real granular piles display SOC has been much debated [5]. A recent experiment on a slowly driven pile of rice [6] has shown that granular piles may display SOC, but the behavior is not universal. For a long-grained rice, power-law distributed avalanches were observed. However, for a rounder rice, the avalanches had a characteristic size. The differences were attributed to the more efficient suppression of inertia and higher dissipation for the long-grained rice.

Several aspects of the surface dynamics and geometry of a driven pile have been studied theoretically. Krug *et al.* [7] studied the surface fluctuations in the one-dimensional local limited model and the corresponding inertially dominated modifications, and concluded that the width of the active zone scaled with the system size, $w \propto L^{\chi}$. The exponents χ were identical for all models, independent of the avalanche behavior. In continuously driven systems, the time correlation function has been used to compare predictions from differential equations to simulation [8]. Recently, differential equations have been proposed for the surface evolution [9,10] and the scaling properties of the time correlation function have been discussed in detail, providing a set of exponents describing the scaling of the front width for models with different driving conditions and interactions. The results from an experiment on the transport properties in rice piles also indicated a scaling relation $w \propto L^{\chi}$, with exponent $\chi = 0.5 \pm 0.2$ [11].

Given the significant theoretical effort to characterize pile surfaces, it is surprising that no detailed experimental studies have been performed. Here, we analyze the geometry and time correlations in a one-dimensional pile of rice for both a self-organized critical system and a noncritical system, and compare with results from the Oslo model sandpile [11].

Experiments.—The rice pile was confined between two glass plates, kept apart by rectangular aluminum rods. Grains were inserted along the left supporting wall from a single seed sowing machine (Övrum, Sweden), dropping 2–3 grains into the system every 7.5 seconds. Before entering the system, the grains were slowed down by a flexible paper flip and inserted at an approximately constant height above the top of the pile. The bottom horizontal rod determined the system size, the distance $\mathcal{L}$ from the left wall to the outlet. Two types of rice grains were used: An elongated, unpolished rice (rice A) and a rounder, polished rice (rice B) (see Ref. [6] for details). The distance between the glass plates was 0.8 grain lengths. Experiments were performed for several system sizes, $L = \mathcal{L}/\delta$, where δ is the grain length. The system sizes $L = 16$, 33, 66, and 105 were used for rice A and $L = 26$, 52, and 104 for rice B.

The time development of the pile was recorded by a CCD camera with 2000×2000 pixel resolution. Pictures were taken at 15 second intervals, corresponding to an addition of 4 to 6 grains. For each picture the borderline between the pile and the black background was extracted. The height $h(x, t, L)$ of the surface at a distance x from the outlet was found as the maximum vertical distance to the surface, removing overhangs in the process. Both the height h and the distance x were measured in units of grain lengths, and time t was measured in units of pictures taken. Each experiment lasted approximately 40 hours corresponding to 10 000 height profiles, except for the largest system of rice A, which had 15 000 profiles.

Profile fluctuations.—Avalanches change the rice pile surface, but only a small zone along the profile is affected. The pile profile fluctuates around an average profile, and the size of the fluctuations characterize the profile

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width. A reasonable definition of the profile width w is the standard deviation of the height, $w(x) = \sigma_h(x, L)$, at a position x averaged over all profiles. The curves describing $\sigma_h(x, L)$ are irregular, and for both types of rice the profile width decreases close to the outlet and increases close to the inlet. In an intermediate region the profile width is approximately constant for a given system size. The average profile width $w = \langle \sigma_h \rangle$ increases with system size (see Fig. 1). The behavior is consistent with a power law $\langle \sigma_h \rangle \propto L^{\chi}$ with $\chi^A = 0.2 \pm 0.1$ for rice A and $\chi^B = 0.3 \pm 0.2$ for rice B. However, the values of $\langle \sigma_h \rangle$ span only the interval from 0.8 to 2.0. These results are therefore not sufficient to establish a power-law relationship, but indicate that the results are consistent with a power-law interpretation.

For the $L = 66$ experiment with rice A, there were recurring problems with the feeding mechanism, and, moreover, the system had initially not reached the equilibrium state. This produced "holes" in the time sequence which invalidated measures of fluctuations and time correlations. We have included the data point here for completeness, but exclude this system from the subsequent analysis.

A similar set of measurements were performed for the Oslo model sandpile [11], which consists of a sequence of columns of height h_i, numbered from the inlet at $i = 1$ to the outlet at $i = L$. Grains are added sequentially at the inlet. A column topples if $z_i = h_i - h_{i+1} > z_i^c$, and transfers one grain to the next column: $h_i \rightarrow h_i - 1$, $h_{i+1} \rightarrow h_{i+1} + 1$. The critical slopes $z_i^c = 1$ or 2 are chosen randomly each time a column topples. Profile widths are shown in Fig. 1 for a narrow range of system sizes. For a large span of system sizes $\langle \sigma_h \rangle \propto L^{\chi^S}$, with $\chi^S = 0.25 \pm 0.05$ [12].

Profile correlations.—The state of the pile is characterized by the height field $h(x, t, L)$. Several dynamical features can be observed in the space-time diagram of Fig. 2: The surface is changed by intermittent avalanches of varying sizes. As a result, there are spatial and temporal correlations in the sequence of profiles. Large steps in the profile appear less frequently than more shallow regions. However, when created, the steps are surprisingly stable. The full dynamics is complicated. Simpler, integrated measures for the state are therefore desirable.

The mass $m(t, L)$ and the potential energy $U(t, L)$ represent different physical characteristics of the state and have also frequently been used to characterize transitions between states. For the rice piles these quantities traced out irregular curves as functions of time with stepwise changes varying several orders of magnitude. The direction of time was evident from the characteristic sawtooth shape of the curves. The temporal evolution of the mass was characterized by the second order time correlation function,

$$C^m(\tau, L) = \langle [m(t + \tau, L) - m(t, L)]^2 \rangle_t, \qquad (1)$$

where the average is over all profiles, and the mass was measured in the number of grains. The time correlation function displayed a consistent behavior for both types of rice and for all system sizes. For small τ, the function increased systematically until τ reached a crossover time τ_m, where the correlation function flattened to a constant value. The crossover time τ_m increased systematically with system size. A data collapse of the form $C^m(\tau, L) = L^{\beta_m} g_m(\tau L^{-\nu_m})$ was therefore attempted (see Fig. 3). For rice B the data collapse was good, and the scaling exponents were found to be $\beta_m^B = 2.0 \pm 0.2$ and $\nu_m^B = 1.25 \pm 0.1$, implying that $\tau_m \propto L^{\nu_m}$. The scaling function g_m is a constant for large arguments and approximately a linear function for small arguments. The data collapse for rice A was not equally convincing, in particular below the correlation time τ_m. However, the crossover times were estimated from the crossover point on the curves, producing the scaling exponents

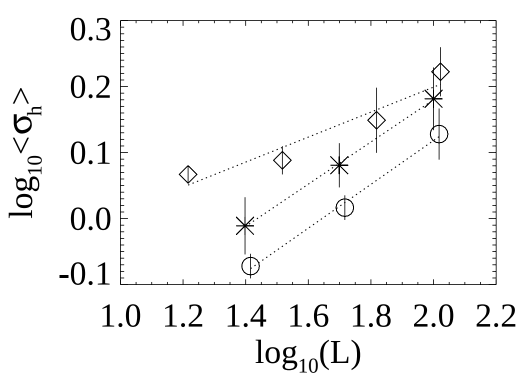

FIG. 1. A plot of the profile width $\langle \sigma_h \rangle$ as a function of L averaged over the central half of the pile for rice A ($\diamond$), rice B ($\circ$), and the Oslo model ($*$). The dotted lines show fits to a power law with exponents $\chi^A = 0.2 \pm 0.1$ for rice A, $\chi^B = 0.3 \pm 0.2$ for rice B, and $\chi^S = 0.30 \pm 0.05$ for the Oslo model.

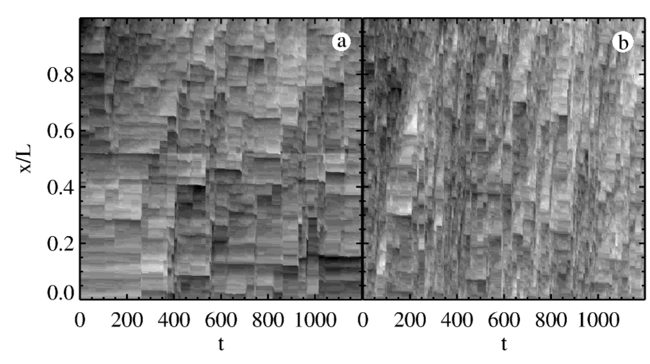

FIG. 2. An illustration of the space-time development of the rice pile system. The gray scale indicates the difference between the height and the average height at a point in space and time. The vertical direction gives the position along the horizontal axis of the pile and the horizontal direction gives subsequent profiles. The inlet is situated at the top of the picture. Panel (a) is for the elongated rice A and (b) is for the more spherical rice B.

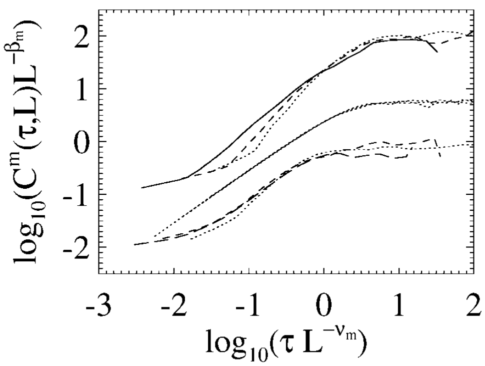

FIG. 3. Data collapse plots of the time correlation functions for the mass of rice A (upper curves) and rice B (lower curves) with increasing dash length for increasing system sizes. The curves for rice B have been shifted by a factor 10^{-1} for clarity. The middle curves show the corresponding data for simulations of the Oslo model for system sizes from 100 to 400. The exponents are $\beta_m^A = 2.2 \pm 0.2$, $\nu_m^A = 1.25 \pm 0.1$ for rice A, $\beta_m^B = 2.0 \pm 0.2$, $\nu_m^B = 1.25 \pm 0.1$ for rice B, and $\beta_m^S = 2.50 \pm 0.05$, $\nu_m^S = 1.25 \pm 0.05$ for the Oslo model.

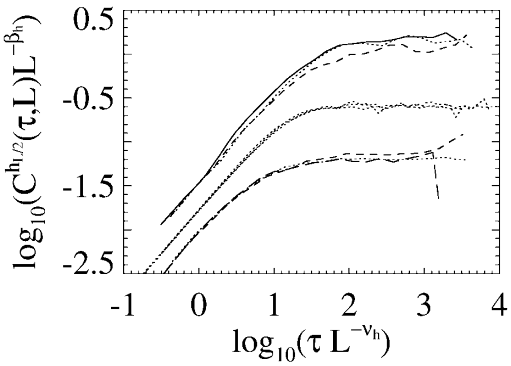

FIG. 4. Data collapse plots of the time correlation function for $h(x = L/2, t; L)$ for rice A (upper curves) and rice B (lower curves) with increasing dash length for increasing system sizes. The curves for rice B have been shifted by a factor 10^{-1} for clarity. The middle curves show the corresponding data for simulations of the Oslo model. The exponents are $\beta_h^A = \beta_h^B = 0.25 \pm 0.1$, $\nu_h^A = \nu_h^B = 0.25 \pm 0.1$ for rice A and B, and $\beta_h^S = 0.50 \pm 0.05$, $\nu_h^S = 1.25 \pm 0.05$ for the Oslo model.

$\beta_m^A = 2.2 \pm 0.2$ and $\nu_m^A = 1.25 \pm 0.1$. The Oslo model displayed a similar scaling behavior with exponents $\beta_m^S = 2.50 \pm 0.05$ and $\nu_m^S = 1.25 \pm 0.05$.

The numerical values of the crossover times were compared for the two types of rice by defining τ_m as the time at which the correlation function had reached half the plateau value. For the largest systems, $\tau_m^A = 592 \pm 30$ (profiles) for rice A and $\tau_m^B = 114 \pm 10$ (profiles) for rice B.

The time correlations in the profile were also measured directly for the height at a distance $x = L/2$ from the inlet using a correlation function of the same form as in Eq. (1). Figure 4 shows a finite size scaling plot of the time correlation function. After a crossover time τ_h, the scaling function changed from an approximately linearly increasing function to a constant. There is a reasonably good data collapse giving the scaling exponents $\nu_h^{A,B} = 0.25 \pm 0.1$ and $\beta_h^{A,B} = 0.25 \pm 0.1$ for rice A and rice B. The Oslo model obeyed a similar scaling function with different exponents: $\beta_h^S = 0.50 \pm 0.05$ and $\nu_h^S = 1.25 \pm 0.05$.

Discussion.—When the rice pile has reached a stationary state, the total mass of a rice pile fluctuates around an average value. The time evolution of the mass is thus similar to a random walk in a potential well. The mass is changed stepwise between profiles $m(t + 1) = m(t) + M_t$ where M_t is the difference between the number of particles added and removed during an avalanche. In the stationary state the average value of M_t is zero. However, there are fluctuations in the mass, and on small time scales we assume that the values of M_t are statistically independent. In that limit the correlation function simplifies to $C^m(\tau, L) = \tau \,\mathrm{Var}(M)$, where $\mathrm{Var}(M)$ is the variance of the changes M_t. For large τ the correlation function is independent of τ, since many large avalanches have propagated through the pile and rendered the profiles uncorrelated: $C^m(\tau, L) = 2\,\mathrm{Var}(m)$. The two behaviors are separated by the crossover time τ_m. A similar argument applies to the time correlation function of the height. This picture is in good correspondence with the observed correlation functions, and is also supported by an interpretation of the power spectrum of the mass, which has a $1/f^2$ behavior for large frequencies, with a crossover to white noise for small frequencies. The crossover frequency is related to the correlation time τ, and therefore depends both on the system size and on the type of rice used.

From the scaling form of the time correlation functions we conclude that the standard deviation of the mass is $\sigma_m \propto L^{\beta_m/2}$ and the standard deviation of the height at $x = L/2$ is $\sigma_h \propto L^{\beta_h/2}$. This confirms the scaling behavior suggested by the direct measurements of the profile width, indicating that the profile width is proportional to a power of the system size $w \propto L^\chi$, with $\chi = \beta_h/2$. The scaling data collapses therefore provide the estimates $\chi^A = \chi^B = 0.125 \pm 0.05$ for the exponent χ.

The scaling form of the time correlation function implies that the crossover time τ_m depends on the system size according to $\tau_m \propto L^{\nu_m}$. The crossover time can be interpreted as the decorrelation time, after which the profile has changed sufficiently to lose all memory of its original state. A simple argument relates the profile width and the crossover time: Since avalanches randomly rearrange the profile, the profile loses all memory of its original state after a large avalanche has propagated through the system. The number of grains that are rearranged and removed are proportional to the number of grains in the active zone. On average, a similar number of grains must be added before another large avalanche can occur. We therefore expect the crossover time to be proportional to the size of the active zone: $\tau_m \propto wL \propto L^{1+\chi}$.

This relation is supported by the data presented here. Simulations of the Oslo model confirm the proposed relation $\tau_m \propto wL$, since $w \propto L^{\chi^S}$ with $\chi^S = 0.25 \pm 0.05$ for the Oslo model. We have also found that, for the Oslo model sandpile, the crossover time is proportional to the time between a particle drops off the system, which again is proportional to the size of the active zone. If the argument holds for the rice pile experiments, the scaling relation $\nu_m = 1 + \chi$ provides the following estimates for the exponents $\chi^{A,B} = 0.25 \pm 0.1$.

The scaling behavior of the crossover time for the height, $\tau_h \propto L^{\nu_h}$ is significantly different from that of τ_m. We expected τ_h to increase away from the inlet, since there are many small avalanches that reach the positions close to the inlet, but fewer reach farther down (See Fig. 2). A measurement of τ_h therefore conveys information on both the correlation between avalanches and the size of avalanches. We have not found a good explanation for the scaling behavior of τ_h, but only observe that the exponent ν_h is approximately the same as χ, and therefore τ_h is proportional to the profile width. However, for the Oslo model sandpile, the two correlation times τ_m and τ_h are equal. Consequently, we believe this scaling to be an important test for the applicability of simulation models.

Conclusions.—Several measures indicate that the profile width of the rice pile scales with system size according to $w \propto L^{\chi}$. The different measures display variations in the estimated values of χ. We consider values obtained from finite size scaling collapses to provide the best estimates, and conclude that the exponents are $\chi^A = \chi^B = 0.2 \pm 0.1$, which contains the spread of the different estimates. This suggests that the profile can be described as a self-affine rough surface with a roughness that does not depend on the criticality of the system. The value of the roughness exponent is close to that found for the Oslo model sandpile [11] for which $\chi = 0.25 \pm 0.05$. The roughness exponent is also similar to that found in a surface dynamics study based on differential equations [10]. However, the scaling of the correlation time for the profile height found for the rice pile differs significantly from that of both sandpile models and differential equation formulations. The scaling behavior of the correlation times is the same for the two types of rice, but the numerical values of the crossover times are almost an order of magnitude larger for the critical system. We find it surprising that no significant difference between the two types of rice were found, since the difference in avalanche behavior is very clear. However, this observation is consistent with the theoretical findings of Krug *et al.* [7], who concluded that both self-organized critical sandpile models and their inertially dominated counterparts produce the same profile geometry. It would therefore be interesting to perform an experiment on the transport properties of rice B in order to establish whether the transport properties depend on the self-organized critical state of the system.

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*Permanent address: The Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, United Kingdom.

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