Comment on "Earthquakes Descaled: On Waiting Time Distributions and Scaling Laws"

Lindman *et al.* [1] have used a nonhomogeneous Poisson process with a modified Omori rate, $r(t) \equiv dn/dt = r_M(1 + t/c)^{-p}$, to model earthquake occurrence. We are going to show that, contrary to claims in Ref. [1], this extremely simple description is incomplete in order to explain Bak *et al.*'s unified scaling law [2].

We generalize Lindman *et al.*'s model by introducing an r-dependent waiting-time probability density of the form $D(\tau|r) \propto r^\gamma \tau^{\gamma-1} e^{-r\tau/a}$, which includes the nonhomogeneous Poisson process of Ref. [1], given by $\gamma = 1$ and $a = 1$ (both parameters linked by normalization). The probability density of the waiting times in the Omori sequence, independent of r, is given by the mixing of all $D(\tau|r)$ [3],

$$D(\tau|r_m) = \frac{1}{\mu}\int_{r_m}^{r_M} rD(\tau|r)\rho(r)dr, \qquad (1)$$

where $\rho(r)$ is the density of rates, $\rho(r) \propto |dr/dt|^{-1} = C/r^{1+1/p}$; μ is the mean rate of the sequence, $\mu \equiv \int r\rho dr$; r_M is the maximum rate, corresponding to $t = 0$; and r_m is the minimum rate, related to the background seismicity level. Note that we have emphasized the dependence on r_m.

Easy to deal with but illuminating is the case $\gamma = 1/p$, which yields

$$D(\tau|r_m) \propto \frac{C}{\mu}\frac{(e^{-r_m\tau/a} - e^{-r_M\tau/a})}{\tau^{2-1/p}}, \qquad (2)$$

where the minimum rate r_m determines the exponential tail of $D(\tau|r_m)$ for large τ, preceded by a decreasing power law with exponent $2 - 1/p$ if $r_M \gg r_m$. For $p = 1$, this is in agreement with the simulation and numerics in Fig. 1 of Ref. [1]; however, it can be shown that the exponent $2 - 1/p$ holds even when $\gamma \neq 1/p$, which is in disagreement with Lindman *et al.*'s claim of a $1/\tau^p$ decay for $p < 1$ and $1/\tau^{\sqrt{p}}$ for $p > 1$.

Nevertheless, this description totally ignores the spatial degrees of freedom, fundamental in Bak *et al.*'s approach. In fact, Bak *et al.*'s approach performs a mixing of waiting times coming from different spatial areas (or cells), which are characterized by disparate seismic rates. In particular, each area will have a different r_m, depending on its background seismicity level. This spatial heterogeneity of seismicity can be described by a power-law distribution of mean rates R, R being the total number of events divided by the total time for a given area (see Fig. 1 and Ref. [3]); if we assume that the minimum rate r_m is directly related to the mean rate of the sequence μ, which in turn is in correspondence with the mean rate in the area R, then $p(r_m) \propto 1/r_m^{1-\alpha}$, and, therefore, the waiting-time probability density comes from the mixing,

$$D(\tau) \propto \int_{r_{mm}}^{r_{mM}} r_m D(\tau|r_m)p(r_m)dr_m, \qquad (3)$$

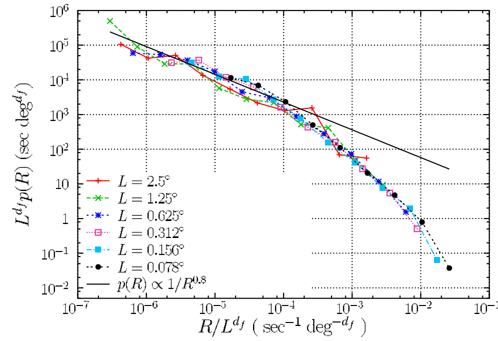

FIG. 1 (color online). Distribution of mean rates R for earthquakes with magnitude $M \geq 2$ in Southern California, dividing the area $(123° W, 113° W) \times (30° N, 40° N)$ in cells of size L and averaging the periods 1984–1992 and 1993–2001. The distributions are rescaled by L^{d_f}, with $d_f = 1.6$. For small R/L^{d_f}, the data are best fit by a density $p(R) \propto 1/R^{0.8}$.

where r_m varies between r_{mm} and r_{mM}. Integration, taking into account that C/μ depends on r_m, leads to

$$D(\tau) \propto 1/\tau^{2+\alpha} \quad \text{for } r_{mm}\tau \ll 1 \ll r_{mM}\tau, \qquad (4)$$

which is in disagreement with Lindman *et al.*'s analysis.

In fact, the power law for long times [Eq. (4)] was established in Ref. [3] for Southern California but without relating it to the spatial heterogeneity of seismicity. The universal value of the exponent $2 + \alpha$, found in Ref. [4] analyzing diverse seismic catalogs, would imply the universality of seismicity spatial heterogeneities. In consequence, Bak *et al.*'s unified scaling law provides a way to measure these properties and is far from being as trivial as suggested by Lindman *et al.*'s approach. Further, Eq. (4) shows that the kink in $D(\tau)$ appears for τ larger than $1/r_{mM}$, which corresponds, for the area of highest seismicity, to the mean of events that are in the tail of the Omori sequence, or in background seismicity, and therefore at the onset of correlation with the main shock.

The model presented here is still too simple for real seismicity but provides a clear visualization of its complexity and the fundamentals of the unified scaling law of earthquakes.

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