A Complexity View on Rain

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Abstract

We show that rain events are analogous to a variety of nonequilibrium relaxational processes in Nature such as earthquakes and avalanches. By identifying rain events as the basic entities of the phenomenon, we find that the number density of rain events per year is inversely proportional to the released water column raised to the power 1.4. This is the rain-equivalent of the Gutenberg-Richter law for earthquakes and the scale-free distribution of avalanche sizes in slowly driven granular piles. The event durations and the waiting times between events are also characterised by scaling regions, where no typical time scales exist. In addition, the accumulated water column displays scale-free fluctuations. All of our findings are consistent with the concept of self-organised criticality, which refers to the tendency of slowly driven nonequilibrium systems to be poised at a critical state without the need of any fine tuning.

Introduction

Rainfall and rainfall-related quantities have been recorded for centuries [1]. All these measurements, however, have the disadvantage of low sensitivity and low temporal resolution. A time series from radar measurements of high precision rain rates with one minute resolution was analysed [2, 3]. The data were collected at the Baltic coast Zingst by the Max-Planck-Institute for Meteorology, Hamburg, Germany [4] using a Micro Rain Radar MMR-2 developed by METEK [5]. The data refer to a height range of 50 m at 250 m above sea level during the period January – July 1999. The detection threshold for rain rates under the pertinent operation parameters was $q_{min} = 0.005$ mm/h. Below this threshold, the rain rate $q(t) = 0$ by definition.

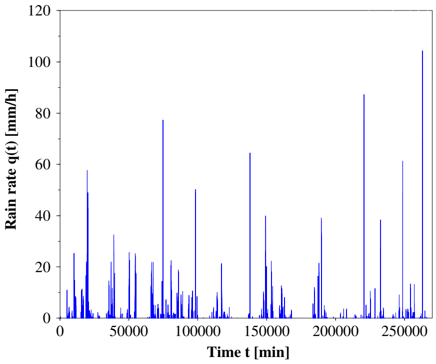

Figure 1: The rain rate $q(t)$ in mm/h, averaged over one minute, versus the time t of occurrence since 01.01.1999 at 00:00.

Results

We define an *event* as a sequence of successive non-zero rain rates and the *event size* is defined as the released water column in mm, $M = \sum_t q(t)\Delta t$, where $\Delta t = 1$ min, that is, the time-integral of the rain rate over an event [2, 3].

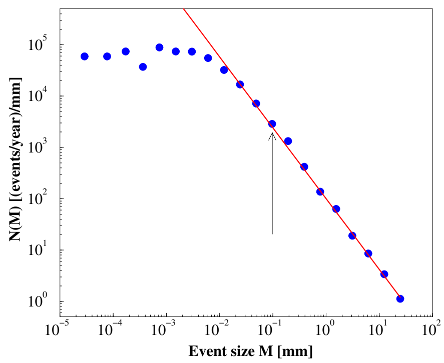

Figure 2: The number density of rain events per year $N(M)$ versus event size M (blue circles). Over at least three orders of magnitude, the data are consistent with a power law $N(M) \propto M^{-\tau_M}, \tau_M = 1.4$ (red line). The arrow indicates the typical sensitivity threshold of a conventional high precision tipping bucket rain gauge. The radar technique is roughly 10,000 times more precise. A considerable fraction $\approx 68\%$ of rain events with size $M > 5 \cdot 10^{-3}$ mm are missed with conventional gauges.

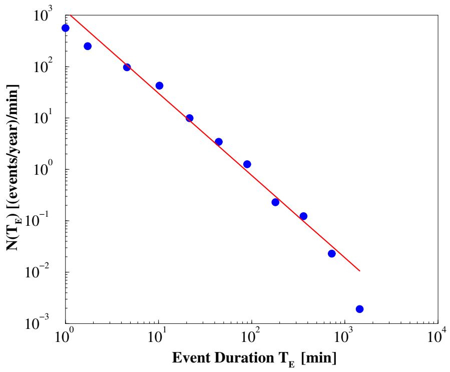

Figure 3: The number density of rain event duration per year $N(T_E)$ versus event duration T_E (blue circles). The data are consistent with a power-law decay $N(T_E) \propto T_E^{-\tau_E}, \tau_E = 1.6$ (red line). The exponent τ_E is larger than the exponent τ_M for the event size distribution, implying a non-trivial relation between event duration and the rain rate during the event. On average, the longer the event, the more intense it is.

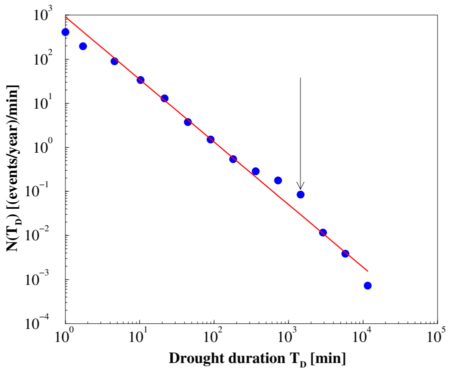

Figure 4: The number density of drought periods per year $N(T_D)$ versus the drought duration T_D (blue circles). The data are consistent with a power-law decay $N(T_D) \propto T_D^{-\tau_D}, \tau_D = 1.4$ (red line). The arrow indicates one day, around which a deviation from pure power-law behaviour can be observed. This is due to the daily meteorological cycle.

In summary, the rain event sizes and durations and the waiting times between events are characterised by scaling regions, where no typical size or time scale exist. The phenomenon of rain is scale free.

Hurst exponent

Water is a precondition for human civilisation and the question of having a reliable water supply is of utmost importance. Hurst considered whether one can design a reservoir that will never overflow nor empty [1]. Assuming the rain is the only supply to a reservoir (or river), the actual water level at time t is determined by

$$X(t,\tau) = \sum_{u=1}^{t} \left(q(u) - \langle q \rangle_\tau\right) \Delta t,$$

where $\Delta t = 1$ min and

$$\langle q \rangle_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} q(t) \Delta t$$

denotes the average outflux in the considered time period τ.

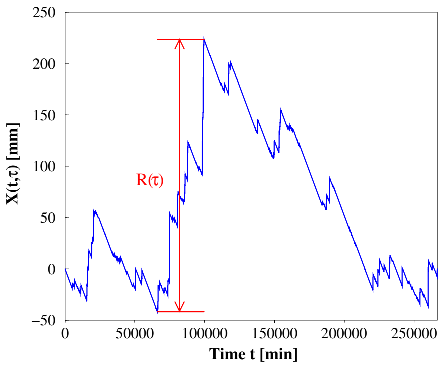

Figure 5: Water level $X(t,\tau)$ in mm in an imaginary reservoir for τ=266,611 min, as derived from Figure 1. During drought periods, a constant decrease in the water level is observed, whilst during rain events the water level increases rapidly. The necessary size of a sufficiently large reservoir is given by the range $R(\tau)$ indicated by the red line.

The water level needed for the reservoir never to empty is given by the range (see Figure 5)

$$R(\tau) = \max_{1 \le t \le \tau} X(t,\tau) - \min_{1 \le t \le \tau} X(t,\tau).$$

One can determine the dimensionless ratio $R(\tau)/S(\tau)$ as a function of τ, where $S(\tau)$ is the standard deviation of the influx $q(t)$ in the period τ. For a random signal $q(t)$ with finite variance $R(\tau)/S(\tau) \propto \tau^H, H = \frac{1}{2}$.

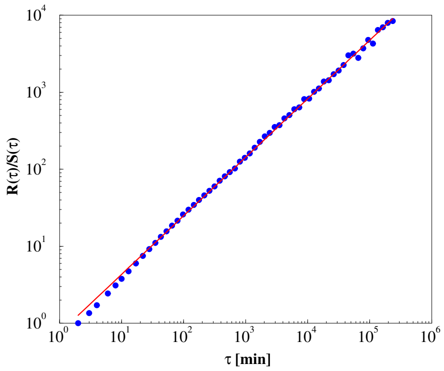

Figure 6: The dimensionless ratio $R(\tau)/S(\tau)$ versus τ (blue circles). The slope of the red line reveals an anomalous Hurst exponent: $R(\tau)/S(\tau) \propto \tau^H, H = 0.76$. The data deviate from the power law below $\tau \approx 10$ min, but no upper limit of the power-law relation is apparent.

Context

Self-organised criticality offers the appropriate framework for dealing with relaxational processes with burst-like behaviour whose statistics are determined by scale-free power laws [6, 7]. The term self-organised criticality refers to the tendency of many systems driven by an energy input at a slow and constant rate to enter states characterised by scale-free behaviour. The statistics of the systems then resemble those of equilibrium systems near the critical point of a phase transition.

Crust of the Earth: A well-known example of such a process is the energy flow through the lithosphere, including the outermost crust of Earth. Tectonic plates are driven at a slow rate by currents in the asthenosphere, the liquid part below the lithosphere, which transports heat by currents. The energy transferred to the plates is intermediately stored in the form of tension until it is finally released in an earthquake. Earthquake statistics follow the Gutenberg-Richter power law that relates the seismic moment, a measure of the released energy, to the probability of such an earthquake [8]. Rather than a typical size with exponentially fewer larger than smaller quakes, scale-free behaviour is observed.

Granular pile: Avalanches in a pile of grains might also display self-organised criticality: when grains are dropped onto a pile, one by one, the pile ultimately reaches a stationary critical state in which its slope fluctuates about a constant angle of repose, with each new grain being capable of inducing an avalanche on any of the relevant size scales [9].

Thus, from the point of view of energy flow through slowly driven nonequilibrium systems, rain is analogous to other relaxational processes in Nature such as earthquakes and avalanches.

System	Atmosphere	Crust of Earth	Granular pile
Energy source	Sun	Convection	Adding grains
Energy storage	Vapour	Tension	Potential
Threshold	Saturation	Friction	Friction
Relaxation	Rain event	Earthquake	Avalanche

Conclusion

New insight into the working of rain can be gained by defining rain events, which can be regarded as energy relaxations similar to earthquakes or avalanches. Taking this perspective, scale-free power-law behaviour is found to govern the statistics of rain over a wide range of time scales and event size scales. Rainfall time series cannot be reproduced by conventional methods of probability theory. To enable anything more than an explicit reproduction of the scaling properties, a deeper understanding of self-organising processes leading to scale free behaviour must be sought. Our findings suggest that rain is an excellent example of a self-organised critical process. From the perspective of self-organised criticality, rain events do not look very different from earthquakes or avalanches. Rain events are relaxations in the sky.

References

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