Brief account of research

I work in the fields of statistical mechanics and complexity with an emphasis on far from equilibrium phenomena. The science of complexity is highly interdisciplinary. It deals with dynamical systems composed of many interacting parts, for example organisms in biology (ecology), grains in granular media, network of agents in economics etc. Methods from statistical mechanics are employed to gain insight into the behaviour of such systems.

The overall objective of the science of complexity is to address why nature is complex, not simple as the laws of physics seem to imply. How, for example, can scale invariance, organisation and pattern formation emerge from simple underlying rules associated with the individual parts? One paradigm that has been proposed is self-organised criticality, commonly illustrated conceptually with avalanches in a pile of grains. If grains are dropped onto a pile one by one, the pile ultimately reaches a stationary critical state with a slope fluctuating about a constant angle of repose, with each new grain being capable of inducing an avalanche on any of the relevant size scales, limited by the system size only.

The applications of self-organised criticality go well beyond granular piles, but the basic picture remains the same: many slowly driven non-equilibrium systems organise themselves, without any external fine-tuning of external control parameter, into a poised state - a critical state - where the susceptibility is very large, diverging with system size. It is possible this is a very general picture for how complexity emerges. However, other principles, yet to be discovered, might be at work.

The seismic system is probably the best candidate for a self-organised critical system in Nature. The slow relative movement of tectonic plates causes stress to build up along the plate boundary. The friction between the plates is finite. Therefore, the strain energy is intermediately stored in the crust of the Earth rather than being relaxed continuously. When the stress exceeds the static friction threshold, an earthquake is triggered that propagates through the crust - much like an avalanche, triggered by a slope exceeding the threshold slope, propagates through the pile. The size of an earthquake is measured by its magnitude m on the Richter scale. The quakes come in all sizes and obey the same statistical law, the Gutenberg-Richter law, which states that the number of quakes with an energy release larger than $s = 10^m$ decays approximately as $1/s^b, b \approx 1$. That earthquakes obey one and the same statistical law, irrespective of size, strongly suggests that large earthquakes and small earthquakes have one and the same physical origin. Compared to the number of earthquakes larger than, say, magnitude $m = 6$, there are ten times as many earthquakes larger than magnitude $m = 5$, and one hundred times as many earthquakes larger than magnitude $m = 4$, and so on. There are many small earthquakes, fewer medium earthquakes and very few large quakes, but there is no typical size of an earthquake. They are scale invariant. Therefore, the crust of the Earth can be viewed as a slowly driven nonequilibrium steady state system with intermittent behaviour that is scale free. As such the crust of the Earth may be an example of a system displaying self-organised criticality.

My research has been an integration of theoretical, computational as well as experimental studies. Statistical mechanics and the science of complexity are widely applicable and of general interest as demonstrated by the range of high-impact journals where the research has been published (e.g., Nature, Physical Review Letters, Journal of Geophysical Research, and Journal of Theoretical Biology) and featured (e.g., Nature, Scientific American, and New Scientist). Below, I comment on some of my major contributions.

Self-organized criticality and $1/f$ noise

The phenomenon of $1/f$ noise, where the strength of the power spectrum is inversely proportional to the frequency, has been observed in many diverse systems such as resistors, the flow of the river Nile, light from quasars, and highway traffic to name a few. $1/f$ noise implies that there is no typical time scale. Scale invariance prevails and hence there are features on all scales from say seconds to years. The temporal signal looks like a mountain landscape with valleys within valleys within valleys and so on. Where does the ubiquitous $1/f$ noise come from? Originally, self-organised criticality was proposed to be an explanation for $1/f$ noise. The spatial and temporal scale invariance would be two side of the same coin. I was instrumental in the first theoretical clarification of the relationship between models displaying self-organised criticality and $1/f$ noise [1,2]. The results showed that a self-organized critical system might, under certain circumstances, display $1/f$ noise. The analytic findings are applicable when the signal is a superposition of elementary signals and the results have been widely used also in the engineering community. Ref. [1] alone has more than 135 citations.

Spring-block model of earthquakes

Soon after the acceptance of plate tectonics in 1967, Burridge and Knopoff proposed a spring-block system to model the build-up and intermittent release of stress along plate boundaries. A fault is represented by a two-dimensional network of blocks interconnected by springs. The blocks rest on a rigid plate and are connected by springs to a rigid slowly moving plate above. An earthquake is triggered when the force on a block exceeds the frictional force resisting the blocks motion. The moving block may induce other blocks to exceed the friction threshold and the earthquake can propagate in the system. Burridge and Knopoff investigated experimentally a one-dimensional model. Together with Olami and Feder, I mapped the Burridge-Knopoff two-dimensional spring-block model onto a coupled map lattice that could be investigated numerically. This simple model contains only the essential basic physics of earthquakes and yet, amazingly, the numerical model reproduces the Gutenberg-Richter law and other phenomenological observations of real earthquake data such as regional variation in the b-value and the spatial distribution of epicentres. Therefore, the Olami-Feder-Christensen model has become a standard model for earthquake within the geophysical community. Also, the Olami-Feder-Christensen model proves wrong an earlier hypothesis that a model can only display self-organized criticality if the relaxing quantity (here the force on the blocks) is conserved. The Olami-Feder-Christensen model has become a research topic in its own right and Ref. [3], where it was introduced, has 548 citations (more than 15% are from articles published in Geophysics Journals) and Ref. [7] discussing its phase diagram has more than 142 citations.

Rice pile experiment and the Oslo model for avalanches in granular media

Rice pile experiment: The metaphor for the concept of self-organized criticality is avalanches in a slowly driven granular pile. However, no controlled experiments existed before I and colleagues from the University of Oslo studied granular flow experimentally in a slowly driven pile of rice. A ricepile was narrowly confined between two glass plates. Because the pile is one-dimensional, we can infer the avalanche sizes of the slowly driven pile by high-precision measurements of the temporal evolution of its profile. We demonstrated that when inertia is suppressed, self-organised criticality occurs. The findings were published in Nature [11] and the experiment is widely seen as classic (288 citations).

Transport is a generic phenomenon. Often, one imagines a static (fixed) medium in which the transport takes place as for example fluid propagating through a porous medium. However, in a generic case, there is a feedback between the medium in which the transport takes place and the material being transported. For example, in the driven rice pile, the avalanches constantly modify the slope of the pile and hence the medium within which they propagate. I and co-workers were the first to point out this important idea and we studied experimentally the transport properties in a critical rice pile. We showed that the average transport velocity of grains through the pile depends on system size, indicating that the profile of the pile looks the same on all length scales [14].

The Oslo Model: These experimental findings were reproduced by a simple one-dimensional computer model of the rice pile experiment, known as the Oslo Model. The Oslo model is the simplest possible model that displays self-organized criticality and therefore it has the status of a canonical model for self-organised criticality in slowly driven non-equilibrium systems much like the Ising model for an equilibrium system displaying a phase transition. Ref. [14] introducing the experimental findings on the transport and the Oslo Model has more than 135 citations and the Oslo Model has become a research topic in its own right. The generality of the Oslo Model has been established since models of earthquakes and interface de-pinning in a quenched random medium belong to the same universality class as the Oslo model.

Biological evolution and punctuated equilibrium The fossil record provides invaluable material for those who seek insight into the history of life on earth, and in the last two decades or so, fossil data have been collected of sufficient quality to allow statistical investigation of large-scale patterns in the extinction and origination record. Two main features observed are the hierarchical organisation of life expressed by the phylogenetic tree and the intermittent nature of evolutionary dynamics, commonly called punctuated equilibrium. Explaining these macro-evolutionary patterns as collective emergent properties of systems with many interacting degrees of freedom, whether these be single individuals or 'species', is an alluring challenge for researchers with a background in statistical physics. In the Tangled Nature model, we introduce only the essential features of a population on micro evolutionary time scales, that is, reproduction, with heredity and variation, and natural selection. Each organism reproduces with a rate that is linked to the individuals' genome and depends on the composition of the living population in genotype space. This is the first individual based model that is able to span from the micro-evolutionary time scale (ecology) to the macro-evolutionary time scale (evolution). The model proves to reproduce qualitatively observed macro evolutionary phenomena. The Tangled Nature model spontaneously displays an emergent structure which we identify with an ecology of coexisting organisms. The dynamics is characterised by long periods of stasis interrupted by bursts of activity which modify the living organisms at all scales, that is, extinctions are local as well as able to affect the entire ecology. Hence, species are emergent structures and extinction, origination and diversity are consequences of co-evolutionary interactions between individuals. Furthermore, the long term behaviour of the dynamics reveals that the ecology becomes gradually better adapted and more complex. The findings are published [19,25,27,29] and are of increasing interest to researchers within complexity as well as theoretical evolutionary biology.

Unified scaling law for earthquakes Earthquakes are a complicated spatio-temporal phenomenon. The number of earthquakes with a magnitude larger than m is given by the Gutenberg-Richter law. In addition to the regularity in the rate of occurrence, earthquakes display a complex spatio-temporal behaviour. The spatial distribution of epicentres is fractal and they occur on a fractal-like structure of faults. Short-range temporal correlations between earthquakes are expressed by Omori's law, which states that immediately following a main earthquake there is a sequence of aftershocks whose frequency decays with time as T^{-1}. This has led to the commonly held belief that aftershocks are caused by a different relaxation mechanism than the main shocks. Together with co-workers, I argued recently that the observed temporal complex behaviour is obviously of dynamical origin. However, the statistics of earthquakes as well as the geometrical fractal structure displayed by the faults and by the spatial distribution of epicentres are also a result of a dynamical process and one might speculate whether it is possible to unify these observations. We showed that the distribution of waiting times between earthquakes occurring in California obeys a simple unified scaling law valid from tens of seconds to tens of years. The short time clustering, commonly referred to as aftershocks, is nothing but the short time limit of the general hierarchical properties of earthquakes. Hence there is no unique operational way of distinguishing between main shocks and aftershocks. In the unified scaling law, the Gutenberg-Richter b-value, the exponent -1 of the Omori law for aftershocks, and the fractal dimension of earthquakes appear as critical indices. The 2002 findings of the MSci project published in Ref. [26] have already attracted a good deal of interest (217 citations) and have been described as seminal.

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Number of citations according to *ISI - web of Science* as of September 2011 is listed in curly brackets. Total 2,444.
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Book

- K. Christensen and Nicholas R. Moloney,
  Complexity and Criticality.

  408 pages. Published by Imperial College Press October 2005.
  URL: http://www.worldscibooks.com/physics/p365.html

  The aim of this book is to introduce the concepts of critical phenomena and explore the common ground between complexity and criticality.

  The word 'complexity' takes on a variety of meanings depending on the context, and its official definition is continuously being revised. This is because the study of complexity is in its infancy and is a rapidly developing field at the forefront of many areas of science including mathematics, physics, geophysics, economics and biology, to name just a few. Institutes and departments have been formed, conferences and workshops organised, books and countless articles written, all in the name of complexity. And yet, nobody agrees on a clear and concise theoretical formalism with which to study complexity. The danger is therefore that complexity research may become unstructured or even misleading. For our purposes, complexity refers to the repeated application of simple rules in systems with many degrees of freedom that gives rise to emergent behaviour not encoded in the rules themselves.

  The word 'criticality', on the other hand, is well defined among statistical physicists. Criticality refers to the behaviour of extended systems at a phase transition where observables are scale free, that is, no characteristic scales exist for these observables. At a phase transition, the many constituent microscopic 'parts' give rise to macroscopic phenomena that cannot be understood by considering the laws obeyed by a single part alone. Criticality is therefore a cooperative feature emerging from the repeated application of the microscopic laws of a system of interacting 'parts'. The phenomenology of phase transitions is well developed and there exists a sound theoretical formalism for its description.

  The book is divided into three chapters. In the first two chapters, we carefully introduce the reader to the concepts of critical phenomena using percolation and the Ising model as paradigmatic examples of isolated equilibrium systems. These systems undergo a phase transition only if an external agent finely tunes certain external parameters to particular values. The underlying theoretical formalism of criticality is carefully explained through the concept of scale invariance, a central unifying theme of the book.

  However, there are many examples in Nature of complexity, that is, the spontaneous emergence of criticality in slowly-driven non-equilibrium systems: earthquakes in seismic systems, avalanches in granular media and rainfall in the atmosphere. Key models of self-organised criticality illustrate how such systems may naturally evolve into a stationary state displaying scale invariance, and analogies are drawn between complexity and criticality.

  Although mathematical methods have been developed to describe complexity and criticality, it is our experience that these methods are unfamiliar to scientists outside the field. Therefore, throughout the book we emphasise the mathematical quantitative techniques available. Our hope is that this book will help students and researchers to treat complexity and criticality more quantitatively.

  The book is based on the lecture notes developed for the Statistical Mechanics course. The target audiences are undergraduate and graduate students and researches in various fields. The book will be self-contained and accessible to readers not familiar with the concepts of complexity and criticality. The text can form the basis for advanced undergraduate or graduate courses, and serve as an introductory reference for researches in various fields. The book includes a generous number of figures, and has an associated website containing solutions to exercises and animations of the models considered. Each chapter is accompanied by exercises, full solutions to which can be obtained by contacting the authors via the book's associated website, http://www.worldscibooks.com/physics/p365.html. On this site, readers will also find animation codes to visualise the behaviour of the models considered.

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